TEACHERS' CLASSROOM PRACTICES IN ADDRESSING LEARNERS' ERRORS IN ALGEBRAIC WORD PROBLEMS: A CASE OF SELECTED SECONDARY SCHOOLS IN ISOKA DISTRICT

By

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(M. Ed-Mathematics Education)

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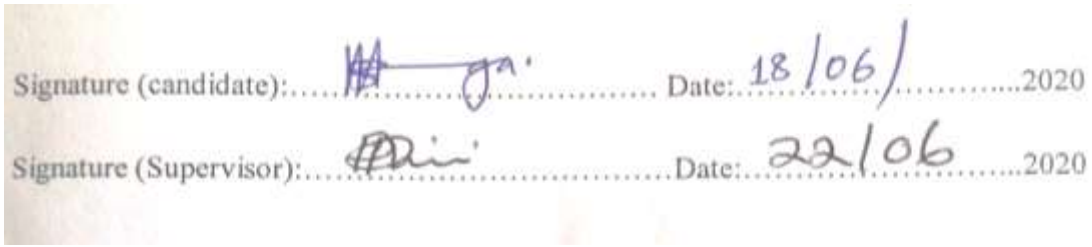
DECLARATION

I, **Stephen Mubanga,** hereby solely declare that the work contained in this dissertation has been composed and written by me and that this work is as the result of my own individual effort. I further sincerely declare that this research has not been previously published for any academic award at any other higher education institution, and that all the sources that I have used or quoted have been indicated and acknowledged accordingly by means of complete references.



APPROVAL

This Dissertation of **Stephen Mubanga** has been approved as fulfilling the requirements for the award of the Degree of Master of Education in Mathematics Education (M. Ed-Mathematics Education) by the University of Zambia.

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ABSTRACT

Good classroom practices in algebraic word problems by teachers at junior secondary school level are crucial if learners are to build a strong foundation in mathematics. Algebra is important as it is a pre-requisite to nearly all forms of mathematics. Many scholars analysed learners' errors in mathematical word problems but did not look at what teachers do in class to identify and correct learners' errors in algebraic word problems, hence the gap. Therefore, the study explored teachers' classroom practices in addressing learners' errors regarding how they engaged grade eight learners in reading; how they probed learners' understanding; and what they did to enhance learners' process skills and encoding abilities in algebraic word problems. Five teachers and twenty five learners (in five focus groups discussions of five members each), at three secondary schools in Isoka district of Muchinga province, Zambia were purposively sampled to participate in the study. The study used a qualitative approach which followed a descriptive case study design. Data collection was done using lesson observations, semi-structured interviews, document analysis and focus group discussions. To capture observations and interviews in totality, video and audio recordings were used respectively. Thematic analysis was used to analyse data. The five performance strategies of solving word problems by Newman namely; reading ability, comprehension ability, transformation ability, process skills and encoding ability guided the analysis of data. Findings revealed that, although teachers engaged grade eight learners in reading, explaining, and solving of algebraic word problems, they did not adequately follow the five steps of solving word problems as outlined by Newman. Teachers read and explained algebraic word problems for the learners and were inconsistent in probing learners' understanding of algebraic word problems. Moreover, teachers did very little to enhance learners' process skills and encoding abilities as even themselves were not consistent in the way they expressed their final answers when solving algebraic word problems. Therefore, teachers did not do enough to discover and correct learners' errors. In view of these findings, it was recommended that teachers should be oriented to Newman error analysis model so that they can apply it in their classrooms when teaching algebraic word problems as it can help them discover and correct learners' errors. Teachers should also ensure that they express final answers accurately and consistently and improve their planning on how to engage learners when teaching algebraic word problems. It was further recommended that teachers should allow learners to take a leading role in their own learning as it can help them discover and address learners' errors.

Key words: *Algebraic word problems, Performance strategies, Reading ability, Comprehension ability, Transformation ability, Process skills, Encoding ability.*

DEDICATION

To

My mother and my late father

Esther Bowa and **Joseph Mubanga**

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ABBREVIATIONS AND ACRONYMS

AIEMS	Action to Improve English, Mathematics and Science
CDC	Curriculum Development Centre
CPD	Continuing Professional Development
DEBS	District Education Board Secretary
ECZ	Examinations Council of Zambia
FGDs	Focus Group Discussions
INSET	In-service Education and Training
JICA	Japan International Cooperation Agency
MESVTEE	Ministry of Science, Vocational Training and Early Education
MOE	Ministry of Education
MOGE	Ministry of General Education
NEA	Newman Error Analysis
NEAL	Newman Error hierarchical Levels
SBCPD	School Based Continuing Professional Development
SMASTE	Strengthening of Mathematics, Science and Technology Education
SSEIT	Self Reporting Emotional Intelligence Test
STEM	Science, Technology, Engineering and Mathematics
TIMSS	Trends in International Mathematics and Science Study
ZAMSTEP	Zambia Mathematics and Science Teachers Education Project
ZPD	Zone of Proximal Development

CHAPTER 1

INTRODUCTION

1.1 Overview

This chapter serves to introduce the study by presenting the background of the study, the statement of the problem, purpose of the study, study objectives, research questions, significance of the study, scope of the study (delimitation and limitations), theoretical and conceptual frameworks, as well as the operational definitions of terms. The chapter ends with the organisation of the study.

1.2 Background of the study

Three aspects motivated me to carry out this study, namely; poor learner performance in mathematics and curriculum revision, teachers' classroom practices in algebra, and my personal experience as a secondary school teacher. I now discuss these aspects in detail.

1.2.1. Poor learner performance in mathematics and curriculum revision

There has been a high failure rate in Zambia at grade 12 in both ordinary level mathematics and additional mathematics, and at grade 9 mathematics for a long time now (Examinations Council of Zambia, 2015, 2016, and 2017). Moreover, government through the Ministry of Education Science, Vocational Training and Early Education (MESVTEE) revised the Zambian Education Curriculum in 2013. This brought about the alignment of topics according to content and process domains. Algebra was one of the six thematic areas of the mathematics content alongside numbers and calculation, geometry, measurement and estimation, relations and functions, and probability and statistics (MESVTEE, 2013). This revised curriculum added topics such as computer and calculator, probability, and matrices at junior secondary school level. Calculus, arithmetic and geometric progression, computer, composite functions, inverse functions, graphs of cubic functions, standard deviation in statistics, and trigonometric equations at senior secondary school level (ECZ, 2016). These topics were added to the already existing bulky content in mathematics at these two levels.

The 2013 revised curriculum was implemented in all the schools in the country starting from 2014. The first cohort of grade nines to sit for national examinations under this curriculum was that of 2015, while the first grade twelves to sit for the national examinations under the revised curriculum was the 2016 intake (ECZ, 2017). Even with the revised curriculum, many learners have continued failing the subject as noted in the chief examiners' reports. According to ECZ (2017, p.27), "Mathematics is still posing a challenge to candidates even under the revised curriculum. It had the highest proportion of failure rate at 41.33 percent in 2016." It was also continuously noted in examiners' reports that the performance of many candidates in mathematics was generally poor and some candidates scored zero at grade 9 and grade 12 final examinations, especially in algebra, and topics which needed candidates to have some understanding of algebra (Examinations Council of Zambia, 2015, 2016 and 2017).

The persistence of poor performance of learners in mathematics and curriculum revision motivated me to explore teachers' classroom practices in addressing learners' errors in algebraic word problems at grade eight level. This is because competences in algebra are essential for learners to handle many topics including those which were added to the curriculum following the curriculum revision of 2013. Moreover, at grade eight level, teachers should ensure that learners acquire conceptual understanding of algebra as it is necessary to help them solve many other complex topics later in their school life. This is so because algebra is a pre-requisite to many other topics. Hence, the need for teachers to build a strong foundation of algebra in learners. In the next subsection I now elaborate on teachers' classroom practices in algebra.

1.2.2. Teachers' classroom practices in algebra

I studied the mathematics examiners' reports of the Examinations Council of Zambia from 2013 to 2018, for both junior secondary school leaving examinations and senior secondary school certificate examinations. The main objective was to find areas where teachers needed to emphasise when teaching algebraic word problems for them to improve performance of candidates in these national examinations. The essence of examiners' reports is to provide feedback to teachers, learners, policy makers, curriculum designers and other stakeholders on learners' performance in the examination with recommendations on how issues identified may be addressed (ECZ,

2014). A repeating theme was eminent in the examiners' reports which indicated that candidates lacked conceptual understanding of algebra to solve algebra and topics that needed the knowledge of algebra. The following are some of the notable excerpts from the reports:

> *The candidate could not expand the bracket to solve the equation. The final answer shows that the candidate lacked understanding of the meaning of the word solve (2016, p.8).*
>
> *Some candidates failed to express the left hand side of a single fraction. Those who could express the left hand side as a single fraction failed to factorise the numerator. Teachers must emphasise to the candidates that the processes of algebra play a role in proving identities (ECZ, 2017, p.62).*

Algebra as one of the six thematic areas of the mathematics content is very important for understanding other content domains, as it acts as a pre-requisite to them. Musonda (2017) observed that algebra is found in all branches of mathematics directly or indirectly. Hence, to improve the results in mathematics at both grades 9 and 12, there was need to explore how algebra is taught at grade eight level, especially algebraic word problems as it is crucial for understanding other topics in mathematics. At that level, teachers must endeavour to build a strong foundation for the learners in mathematics through appropriate teaching of algebraic word problems as it aids in understanding other concepts in the subject. According to Usiskin (2004), competence in algebra is imperative for understanding any mathematics. "A child who learns mathematics without a good understanding of algebra can be compared to 'a child immersed under water in order to teach her to swim" (Ramirez, Yuen and Ramey, 1991, p.3). Lack of competence in algebra is regarded as one major barrier to learning mathematics for many children because it is the language in which mathematics is spoken about and written (Mhakure, Jacobs and Julie, 2014).

Moreover, the National Policy on Education clearly indicates that learners who proceed to grade eight need to be helped in many areas of learning which include their reading and reasoning as well as calculating abilities. The following is a notable extract from the educational Policy document:

> *Due to many factors that adversely affect the quality of education, many of those who proceed into grade 8 are not adequately equipped with essential reading, writing and mathematical skills. Their understanding of the physical*

> *and social sciences is minimal. Much of their knowledge consists in an assemblage of facts that have been memorised but their ability to deal intelligently with these facts is very limited. They may have had little experience in the use of books and little opportunity for self- expression in creative writing or in speech. Generally they are very eager to learn, but may lack the skills and techniques that will enable them to do so. Hence much of the initial grade 8 teaching needs to be directed towards remedying these defects. In particular, language, mathematics and science needs special attention (Ministry of Education, 1996, p. 33).*

The above statement from the national policy on education is meant to encourage teachers to teach grade eights in a diagnostic way. In this way, learners' current abilities can be established, their errors regarding reading, reasoning and writing can be identified and addressed through the provision of appropriate mediation. Hence, my interest to explore whether such practices are being put into practice by teachers when teaching algebraic word problems to grade eights. The importance of such an approach of teaching mathematics cannot be overemphasised as it enables learners acquire conceptual understanding of topics especially at foundational levels such as grade eight where learners need to build a strong foundation in order to understand complex concepts later in their school life.

In the quest to improve performance in mathematics, the government through the Ministry of General Education and cooperating partners introduced a number of programmes or interventions such as Zambia Mathematics and Science Teachers Education Project (ZAMSTEP), Action to Improve English, Mathematics and Science (AIEMS), and Strengthening of Mathematics, Science and Technology Education (SMASTE) among others. According to Carmody (2004), ZAMSTEP a programme which was initiated in 1980 was meant to equip the diploma holders with pedagogical knowledge so that they could confidently and efficiently teach mathematics and science in secondary schools. This intervention was mainly concerned with improving the instructive skills and knowledge of teachers. It became necessary in my study to establish teachers' instructive skills and knowledge in teaching algebraic word problems at grade eight level.

MOE (2001), indicated that in 1994 the Ministry of Education initiated a project which was known as 'Action to Improve English, Mathematics and Science' (AIEMS). This project was designed to improve the quality of education through the development of

sustainable, decentralised system of INSET for teachers in primary and secondary sectors. Modules were developed as training materials and the cascade model was effected for the purpose of having a multiplier effect. Textbooks in English, Mathematics and Science, Science kits as well as Mathematics kits were equally distributed to all secondary schools.

Moreover, Mubanga (2012) stated that the Japan International Cooperation Agency (JICA) Strengthening of Mathematics, Science and Technology Education (SMASTE) School Based Continuing Professional Development (CPD) through Lesson Study in Zambia started in 2005 in the period when JICA favoured implementing comprehensive crosscutting aid on a country-specific basis with a view toward carrying out effective and efficient aid. All these intervention programmes were aimed at improving learner performance in key subjects such as mathematics by strengthening teacher performance and skills. However, even with interventions such as the ones outlined above and many others not outlined here, learners have continued failing mathematics. Moreover, regarding issues in mathematics education in Zambia, Nakawa (2011, 2012) and Nonaka (2013) indicated that learners in Zambia have very low performance due to teachers' low competence and their limited views on mathematics lessons. As I focus on teachers' practices in the teaching of algebraic word problems to grade eight learners, their competences and views on mathematics lessons will come to the fore.

The fact that learners have continued showing poor understanding of algebraic word problems and failing to apply the knowledge of algebra to solve other problems, calls for a serious investigation. "One way of trying to find out what makes algebra difficult is to identify the kinds of errors students commonly make in algebra and then to investigate the reasons for these errors" (Booth, 1999, p. 299). Moreover, understanding algebra is one of the ten key competencies outlined in the revised mathematics syllabus which learners must acquire (Ministry of Education, Science, Vocational Training and Early Education, 2013).Thus learners' poor background in mathematics in general and algebraic word problems in particular despite interventions to strengthen teacher performance and skills motivated me to explore teachers' classroom practices in addressing learners' errors in algebraic word problems at grade eight level.

When I looked at the topics which were included in the mathematics curriculum after curriculum revision of 2013 (as outlined under section 1.2.1. above) for both junior and senior secondary levels, I also realised that there was need for teachers to build a strong foundation for learners at junior secondary school level in algebraic word problems in order for them to comprehend more complex topics in future. Hence the need to understand teachers' classroom practices in addressing learners' errors in algebraic word problems by focusing on teachers teaching grade eights. In the next subsection, I now elaborate on my personal experience as a secondary school teacher for researching on teachers' classroom practices in addressing learners' errors in algebraic word problems at grade eight level.

1.2.3. My personal experience

The year 2009 was a year of great change in my life. In that year I graduated from the college and began teaching in a private sector on the Copperbelt. I thought teaching mathematics was a very easy task, and I was wrong as it required a lot of time to interact with learners, discovering their errors and challenges in the subject and helping them out. This gave me an opportunity to interact with learners in urban areas. In October the same year, I was employed in the Zambian civil service under the Ministry of General Education and sent to a remote upper basic school in Northern province (now part of Muchinga province). After serving at that station for four months, I was transferred to a nearby secondary school. This also presented another opportunity for me to interact with learners in rural areas and appreciate their challenges in mathematics especially in word problems.

Through the interactions I had with learners in both urban and rural areas, I came to realise that learners found it difficult to solve mathematical problems which were expressed in words. At times I tried to test my learners' problem solving abilities by giving them the same question, one expressed symbolically and the other one expressed in words. To my surprise, my learners answered correctly the questions expressed symbolically and had challenges in answering the ones expressed in words. For example, *"Find x in, $2x - 5 = 11$"* and "*Twice the number minus five equals eleven, find the number"*. Learners could find it easy to solve the first one than the second one, as the first one seemed more direct to them than the second one where

they were required to understand the words first and then express it symbolically. This is contrary to the findings of the study by Carraher and Schliemann (1985) who observed that Brazilian street children could perform mathematics when making sales in the streets but were unable to answer similar problems presented to them in a school context. Hence, the need to explore teachers' classroom practices in addressing learners' errors in algebraic word problems. This is important as it enables learners attain conceptual understanding of concepts. Once learners attain conceptual understanding of concepts, they can easily transfer the knowledge between these two contexts, especially that algebraic word problems link mathematics in the classrooms to the real life situations.

Through such interactions, I also came to observe that in urban areas most of the learners could read algebraic word problems accurately but in most cases failed to answer correctly the same questions they read. There were a number of them who could struggle to read too. This made me to reflect on my classroom practices and also to conclude that reading a mathematical word problem does not necessarily guarantee understanding of the problem. I therefore, would spend time asking learners their understanding of given algebraic word problems. Learners at junior secondary school faced a lot of challenges in explaining the algebraic word problems and made a lot of errors when answering. In remote and rural areas unlike in urban areas, I noted that the challenges of solving algebraic word problems started at the reading stage for most of the learners at junior secondary school level. I came across a number of learners who were incompetent at reading at that level and in some cases even at senior secondary school level, unfortunately. I therefore faced challenges in making such learners develop competences in solving algebraic word problems. That is when I realised that I needed to explore teachers' practices necessary for teaching algebraic word problems for the learners to develop conceptual understanding.

In the year 2011, I was transferred from a remote school to a rural school in the same province. At this station, the learners' challenges were similar to those I taught in other schools. At this school I was appointed as the School In-service Coordinator (SIC) to spearhead the in-service training activities for the teachers in the school. The position which I held for eight years at the time of this study. The position exposed me to teachers' classroom practices through teacher observations when conducting School

Based Continuing Professional Development (SBCPD). At times I acted as Head of Mathematics department and had a privilege of observing teachers as per routine of the school. As a department, we also conducted lesson study cycles where planning was done collaboratively, one teacher could present the lesson to the class while others observed and took down their observations. After that there was usually a discussion regarding strengths and weaknesses of the lesson as per lesson study cycle guidelines. Through these teacher observations, I saw the need to understand teachers' classroom practices since they significantly contributed to learners' errors and challenges in understanding mathematical concepts, especially word problems. Hence, poor learner performance in mathematics and curriculum revision, teachers' classroom practices in algebra, and my personal experience as a secondary school teacher motivated me to embark on this study. The section that follows highlights the statement of the problem in view of the concerns outlined above.

1.3 Statement of the problem

Mathematics is one of the compulsory subjects in the country because of its importance in life (Cockcroft, 1982). Despite that, the subject has been recording poor results at all levels in the country, especially at grade 12 school certificate examinations (ECZ, 2016, 2017). More efforts have been made to improve the results in mathematics, some of which are aimed at strengthening teacher performance and skills such as ZAMSTEP, AIEMS, SMASTE, and School Based Continuing Professional Development (SBCPD), as typical examples (MOE, 2001; Carmody, 2004; Mubanga, 2012; MESVTEE, 2013). However, despite all these efforts, the subject has continued recording poor results (ECZ, 2015, 2016, 2017). One area where learners fail badly is in algebra and topics where the knowledge of algebra is required such as calculus, trigonometry, linear programing, and many others. Some studies have been done in order to correct learners errors in various topics in mathematics including algebra (Kembitzky, 2009, Bush, 2011, Chauraya and Brodie, 2013, Mbewe, 2013). However, these studies only analysed learners' errors of written work in word problems and other topics in mathematics without looking at teachers' role in the classroom regarding learners' errors identification and correction. This motivated me to embark on an empirical investigation of exploring teachers' classroom practices in addressing learners' errors in algebraic word problems at grade eight level.

1.4 Purpose of the study

The purpose of this study was to explore what teachers do to identify and correct learners' errors when teaching algebraic word problems so that learners' conceptual understanding of the topic is enhanced.

1.5 Study objectives

1.5.1 General objective

The general objective of this study was to explore teachers' classroom practices in identification and correction of learners' errors in algebraic word problems.

1.5.2 Specific objectives

The specific objectives of this study were as follows:

i. To examine how teachers engage learners in reading algebraic word problems in their classrooms.
ii. To assess how teachers probe learners' understanding of algebraic word problems.
iii. To determine what teachers do to enhance learners' process skills and encoding abilities in algebraic word problems.

1.6 Research questions

The study was guided by the following research questions:

i. How do teachers engage learners in reading algebraic word problems?
ii. How do teachers probe learners' understanding of algebraic word problems?
iii. What do teachers do to enhance learners' process skills and encoding abilities in algebraic word problems?

1.7 Significance of the study

The importance of this study is that it focused on teachers' classroom practices in addressing learners' errors in algebraic word problems. It explored how teachers engaged learners in their own learning and how they identified and addressed learners' errors in algebraic word problems. Hence, the study could contribute to the body of knowledge in mathematics education in five ways: Firstly, it may benefit teachers of mathematics about good classroom practices necessary to improve learners' performance in algebraic word problems. Secondly, it may benefit teacher educators concerning teachers' pedagogical and social skills that prospective teachers need.

Thirdly, it may inform curriculum designers regarding effective curriculum implementation guidelines to teachers of mathematics. Fourthly, the study may inform policy makers on what aspects of teaching to consider when making educational policies regarding teachers' pedagogical and social skills. Fifthly, the findings could contribute significantly to research in mathematics education on teachers' classroom practices in addressing learners' errors in algebraic word problems, locally, globally and further contribute to the improvement of results in mathematics and STEM education.

1.8 Scope of the study

1.8.1 Delimitation of the study

Delimitations show how the study is narrowed in scope (Creswell, 1994). This study was restricted to three government secondary schools of Isoka district of Muchinga Province, Zambia. I used three secondary schools in order to have an in-depth understanding of teachers' classroom practices regarding learners' errors identification and correction in algebraic word problems. In addition, the study was restricted to teachers who were teaching grade eights at that time. According to Charness, Kelly, Bosman and Mottram (2001), learners exhibit a lot of errors in early grades.

1.8.2 Limitation of the study

Limitations are the potential weaknesses in the study which are out of control of the researcher (Simon, 2011). Hence, this section outlines factors that could have affected the outcomes of this study in some way. The study did not look at teachers' classroom practices in addressing learners' errors in algebraic problems expressed symbolically or other components of algebra as espoused in the grade eight syllabus. Doing so would have enriched the findings of the study in that diverse contexts of algebra would have been represented in relation to teachers' practices. However, that was not done because doing so could have broadened the scope of the study. The small sample size used in the study also restricted the generalisation of the findings. In the next section, I now discuss the theoretical and conceptual frameworks which guided this study.

1.9 Theoretical framework

This study was informed by the sociocultural theory of development by Vygotsky (1978), particularly the Zone of Proximal development (ZPD). According to Vygotsky

(1978, p.86), "The Zone of Proximal development is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers." Actual developmental level indicates a learner's level of mental development at a given time. It indicates the functions that have already matured in the learner. This implies that teaching of new concepts should consider the learner's readiness and prior mathematical understanding (Gullen, 2000). The teacher's role in teaching mathematics in the ZPD becomes one of decisive instruction, a mediator of activities and substantial experiences allowing the learners to attain their ZPD or their potential developmental level (Blanton, 1998; Rueda et al, 1992). According to Donald, Lazarus and Lowlwana (2003, p.3), "Constructivist theorists believe that knowledge is not passively received but actively constructed cognitively and through socialisation." The theory assumes that learning can only be evident when individuals themselves act on the knowledge or take part fully in the learning process (Gullen, 2000). Figure 1 is a diagrammatical illustration of the ZPD.

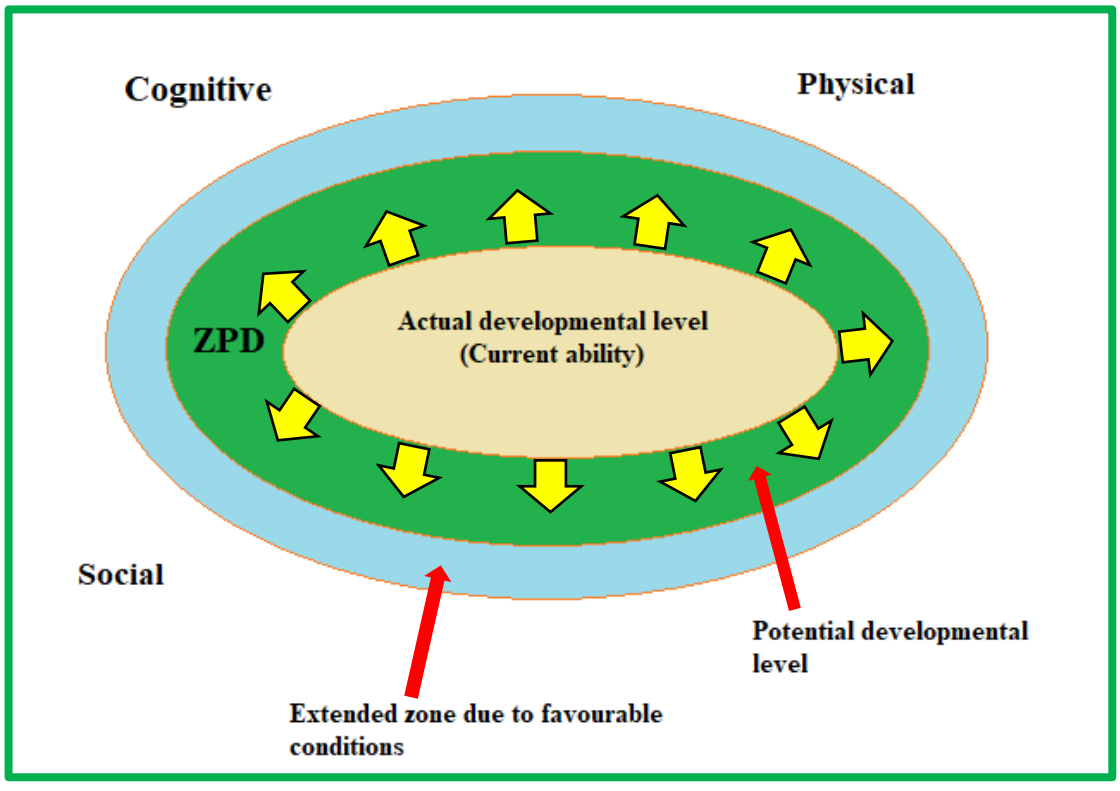

Figure 1: *The ZPD (Source: Adapted from McLeod, 2012)*

From Figure 1 above which is a diagrammatical representation of the ZPD as described by Vygotsky (1978), it can be noted that by engaging learners in reading, probing their understanding through asking them to explain the meaning of algebraic word problems, and by probing their process skills and encoding abilities, teachers can determine the actual developmental level (current ability) of individual learners. In other words, teachers can determine what learners can do unassisted and what they can do with assistance from the more knowledgeable others or in collaboration with more capable peers.

By doing that, teachers can identify learners' errors which can then be corrected by providing the appropriate mediation so that learners can reach their potential developmental level in the ZPD. Mediation is central to Vygotsky sociocultural theory (Williams and Burden, 1997). Mediation according to Vygotsky (1978) refers to the part played by other important people in the learners' lives, people who enhance their learning by selecting and shaping the learning experiences presented to them. The secret of effective learning lies in the nature of the social interaction between two or more people with different levels of skills and knowledge (O'Neil, 2011). This involves helping the learner to move into and through the next layer of knowledge or understanding (Williams and Burden, 1997).

Moreover, if that is done consistently when teaching algebraic word problems, the learners' potential development level can eventually become their actual developmental level. This can therefore enhance learners' problem solving skills to tackle even more challenging tasks involving algebraic word problems and reach more potential developmental level. Through that, learners can acquire conceptual understanding of algebraic word problems and that can enable them to understand algebra related topics in mathematics easily.

Glasersfeld (1987) stated that in the ZPD the teacher will try to maintain the view that the learners are attempting to make sense in their experiential world and he or she will be interested in learners' errors and indeed every instance where learners deviate from the teacher's expected path because it is these deviations that throw light on how the learners at that point, in their development are organising their experiential world. Jaworski (1994) added that in order to help a learner, the teacher has to understand

something of a learners' conceptual structures, not just affect the learners' responsive behaviour. Therefore, it is important to probe learners' reading and understanding of algebraic word problems, as it helps in reviewing learners' errors which in turn the teacher can correct through mediation.

1.10 Conceptual framework

The conceptual framework below on Figure 2 gives the understanding of the main assumptions of teachers' classroom practices in addressing learners' errors in algebraic word problems. When developing the conceptual framework, literature was integrated into the conceptual framework. Literature review explained the theoretical framework broadly in terms of teachers' classroom practices in learners' errors identification and correction in particular. The theoretical framework informed the conceptual framework by enhancing teachers' practices in error identification and correction through mediation. In Figure 2, the first arrow shows that teachers' classroom practices and learners' errors in algebraic word problems influence each other. That is, teachers' classroom practices should aim at identifying and correcting learners' errors in algebraic word problems, and learners' errors also do determine the kind of classroom practices teachers need to employ. The second set of arrows indicate that learners' errors may be seen through their reading abilities, comprehension and transformation abilities as well as their process skills and encoding abilities. The third set of arrows indicate that the learners can show abilities from any of those three stages indicated. For example, a learner may not be able to show the reading ability verbally, but if he or she can show some understanding of the word problems through writing, it is assumed that reading ability is implied even though not verbalised (Newman, 1977). The fourth set of arrows indicate that once the errors are noted in the learners' abilities stated, an appropriate mediation can then be applied by the teacher in order to address those errors. It is important to note that the nature of errors and mediation influence each other directly. The last arrow shows that with the appropriate mediation by the teacher in the correction of learners' errors, they can attain conceptual understanding of algebraic word problems. It also indicates that mediation and conceptual understanding of algebraic word problems do influence each other.

Hence, Figure 2 shows that teachers' classroom practices must be directed towards identification and correction of learners' errors in algebraic word problems. Learners' errors can be seen when the teachers give them chance to read algebraic word problems. These errors may also be noted when teachers probe learners' comprehension and transformation abilities as well as their process skills and encoding abilities. Moreover, that can also help teachers to establish learners' prior knowledge regarding what they can do on their own and where they need some help to do what they are expected to.

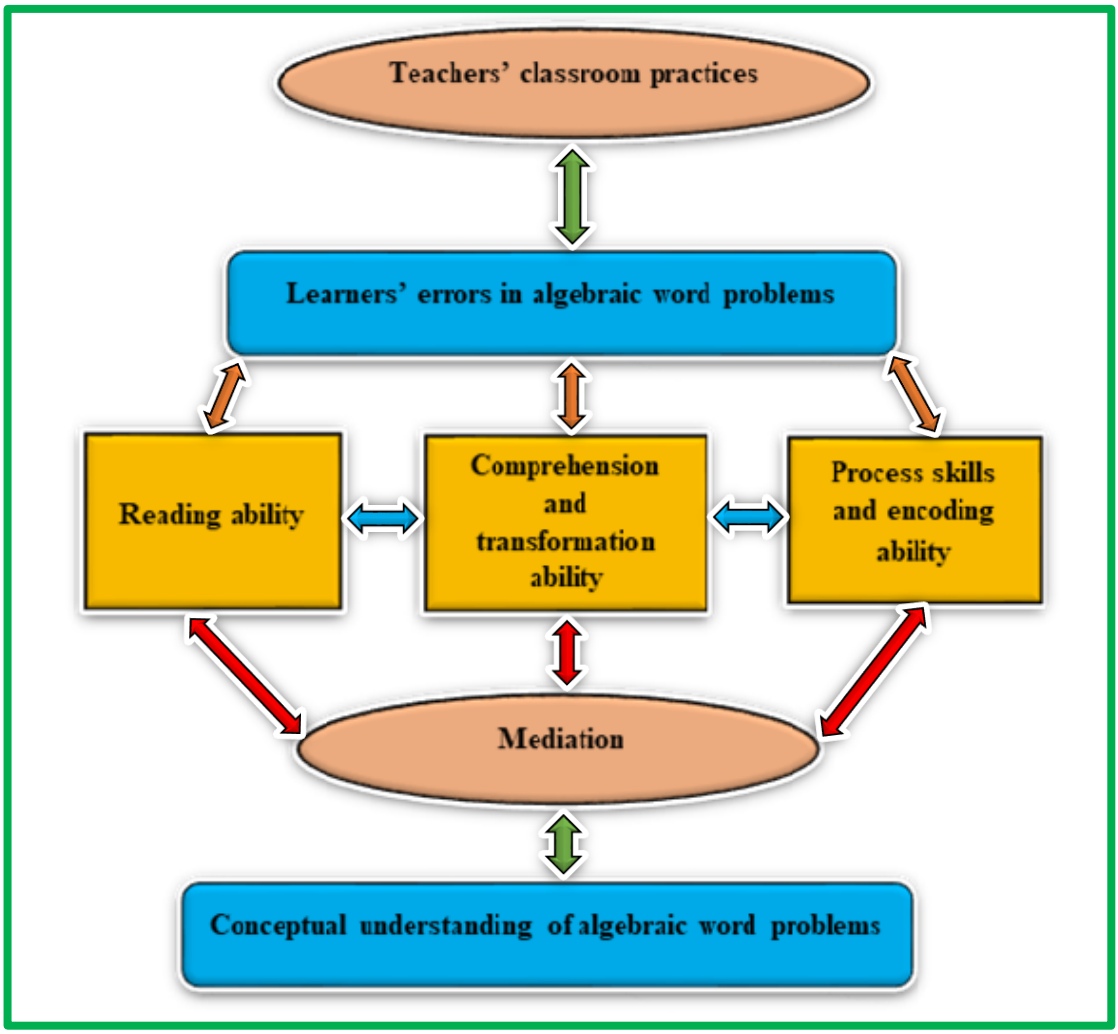

Figure 2: *Conceptual Framework (Source of some concepts: Newman, 1977)*

Newman (1977), outlined some criterion for error causes as, reading ability, comprehension ability, transformation ability, process skills and encoding ability. The ideal way of teaching word problems according to Newman (1977) is by giving opportunities to the learners to show their abilities in reading word problems,

explaining the meaning of word problems in their own words, stating what is required to do to gain a solution, performing the actual processes of getting the solution to the question, and then writing down the answer to word problems correctly and accurately. However, "If the pupil cannot explain the meaning of the question in his own words, but is able to perform the task correctly, it is likely that comprehension has occurred even though it was not verbalised" (Newman, 1977, p.34).

Errors can occur at any stage while solving word problems and the use of Newman's procedure has been established as a means to identify, categorise and analyse learners' errors, with a view towards further action by the teacher (Chinen, 2008; Clements, 1982; White, 2005). Therefore, teachers must endeavour to establish what sort of errors learners make in algebraic word problems, and at what stage, as that can give them a clue of what appropriate mediation they can employ to correct the errors identified, in order for conceptual understanding to be attained by the learners.

1.11 Operational definition of terms

Algebra: the part of mathematics in which letters and other general symbols are used to represent numbers and quantities in formulae and equations expressed in words.

Algebraic word problem: algebra story problems that requires learners to translate the concrete to the abstract and the abstract to the concrete in a given context.

Classroom practices: this refers to the teachers' ways of teaching and the strategies used when teaching algebraic word problems.

Errors: in this study an error shall be regarded as a mistake in the process of solving algebraic word problems procedurally or by any other method.

Reading ability: the ability of learners to read algebraic word problems correctly.

Comprehension ability: the ability of the learner to explain the meaning of the algebraic word question at hand in his or her own words.

Transformation ability: the ability of the learner to select the mathematical processes which are needed to gain a solution for the algebraic word problem at hand.

Processing skills: the ability of the learner to explain the actual performance of mathematical operations necessary for answering algebraic word problem at hand.

Encoding ability: the ability of the learner to answer correctly and express himself or herself in writing of the final answer in algebraic word problems.

1.12 Organisation of the dissertation

Chapter 1 introduced the study by giving the background of the study. The motivation of the study being: poor learner performance in mathematics in general, and algebra in particular and curriculum revision; teachers' classroom practices in algebra; and personal experience. The chapter also outlined some key items such as the statement of the problem, purpose of the study, study objectives and their research questions, the significance of the study, scope (delimitation and limitation), the theoretical and conceptual framework and lastly the operational definitions of terms.

Chapter 2 outlines a review of related literature to the problem under exploration. Literature is presented under the following sub-headings: Importance of algebra; Criterion for error causes or performance strategies; Learners' errors in algebra; Teachers' classroom practices and learners' errors; studies on algebra and related topics, and gaps identified. The chapter concludes with the summary in tabular form showing the themes, key issues and their relation to the current study.

Chapter 3 provides the methodology which includes, the research paradigm, research design and approach, study area, study population, study sample, sampling techniques, data collection methods and instruments, data collection procedures and time line, data analysis, credibility and trustworthiness of the research, and ethical considerations. Lesson observation schedule was accompanied by video recordings. Semi-structured interviews, document analysis, and FGDs were used to supplement the lesson observations.

Chapter 4 presents the analysis of qualitative findings. Newman's (1977) performance strategies namely, reading, comprehension, transformation, process skills, and encoding abilities were used for both vertical (within case analysis) and horizontal

(across case analysis). However, the document only reflects the horizontal analysis. The chapter ends with a summary of the findings.

Chapter 5 provides the discussion of the findings presented in the preceding chapter (chapter 4) in line with the research objectives. The findings are further discussed in view of the literature reviewed and the theoretical foundations that informed the study.

Chapter 6 gives the conclusion and recommendations based on the findings. The conclusion summarises the study while recommendations provide suggestions to inform policy, practice and further research in mathematics and algebra education.

CHAPTER 2

REVIEW OF RELATED LITERATURE

2.1 Introduction

This chapter presents a review of related literature that informed this study. This is important as it links the study to other studies or researches that have been done in the past in this field (Creswell, 2009). Hence, the chapter explores and acknowledges the works of other researchers which are related to teachers' classroom practices, learners' errors in mathematics in general, and algebraic word problems in particular. The chapter begins with an overview of the importance of algebra, then discusses the criterion for error causes or performance strategies, the next sections looks at learners errors in algebra, followed by teachers' classroom practices and learners' errors, the chapter then discusses studies on algebra and related topics and gaps identified. Finally, the summary of the chapter is presented. Being aware of the importance of algebra, the criterion for error causes or performance strategies, learners' errors, and teachers' classroom practices' with regards to learners' errors is crucial as that can help teachers to identify areas where they need to improve and those areas which are problematic for both learners and teachers, thereby helping them to teach effectively. In research, this focus helped me to identify areas which are difficult for learners and where to base my study by looking at how teachers teach algebraic word problems to address those difficulties.

2.2 Importance of algebra

Proficiency in algebra is the fundamental mathematical requirement for learners to proceed into higher-level studies in Science, Technology, Engineering and Mathematics (STEM) courses (Adelman, 2006). Algebra constitutes 40% of the Trends in International Mathematics and Science Study (TIMSS) Mathematics assessment and most importantly, algebra serves as an access towards learning other areas of Mathematics (Ely and Adams, 2012; Usiskin, 2004). In addition, other scholars have argued that algebraic word problem solving is a most useful tool, because it simulates the knowledge and skills that incorporate the application of mathematics in real life situations. The more algebra proves its importance, the more

algebraic word problems typically show they deal with using mathematical ideas in tangible world conditions (Haghverdi, Semnani and Seifi, 2012). Moreover, Usiskin (2004), indicated that there are so many reasons for learning algebra. Some of these are: algebra is a gatekeeper as it is a requirement for further studies in colleges and universities; algebra is the language of generalisation. For example in English, Usiskin further indicated that algebra is the language of relationships among quantities. It is also a language for solving certain types of problems.

Algebra is also the study of structures with certain properties and also shows that our universe possesses order. Most importantly is that algebra is a prerequisite for virtually all other mathematics. Thus without knowledge of algebra, a person loses control over parts of his or her life. This simply means the person relies on other people to do these things for him or her. Hence, the person is more likely to make unwise decisions, may not understand many things in everyday ideas, and he or she may not understand or comprehend ideas discussed in other disciplines such as Chemistry, Physics, Earth sciences, Economics and many others (Usiskin, 2004). It can be seen from the above literature that algebra is useful in many contexts of life. This implies that algebra must be taught in a way that enables learners to have deeper understanding of the topic as it is a pre-requisite to virtually all other mathematics. Hence my interest in establishing teachers' classroom practices in addressing learners' errors in algebraic word problems.

2.3 Criterion for error causes or performance strategies

It is not the revelation of a new fact to state categorically that large numbers of children are experiencing severe difficulties in mathematics and actually this is the concern of both teachers and parents (Newman, 1977, 1983). Newman (1977) came up with a hierarchy of "performance strategies" which need to be applied to solve written mathematical tasks successfully. According to her, failure to apply the appropriate "performance strategy" at the appropriate time would most likely prevent a learner from progressing to the next strategy, and therefore prevent him or her from gaining the correct answer. Newman also referred to these strategies as "Criterion for Error causes" as they could be used to identify learners' errors when teaching or solving word problems in mathematics. These performance strategies or criterion for error

causes are: Reading ability, Comprehension ability, Transformation ability, Process skills, and Encoding ability among others (Newman, 1977, 1983). This sequence of solving word problems is also referred to as the 'Newman's Error Analysis Model' (NEA model) (Mukunthan, 2013).

In reading ability, the learner must show competence in word and symbol recognition while in comprehension ability the learner must explain the meaning of the question in his or her own words clearly illustrating word and symbol recognition. Reading a mathematical problem before solving is essential as it helps solvers to understand the problem at hand (Wertheimer, 1959). Kumar (2018) added that, reading word problems before solving helps to see if learners can decode correctly the key words and symbols used in the mathematical problem. Polya (1957) stated that it is foolish to answer the question which you have not read and you do not understand. Hence, it is important to probe learners' reading and comprehension abilities before asking them to solve the problems. Transformation ability has to do with the learner being able to decide which mathematical process or processes are needed to gain the correct answer. Process skills have to do with the ability of the learner to use the mathematical skills necessary for the item and Encoding ability deals with how the learner writes the final answer which includes correct presentation of the final answer, correct spellings of words and correct use of symbols (Newman, 1977, 1983).

Moreover, Newman indicated that if the learner is unable to explain what the question means in his or her own words, faulty comprehension should not be assumed unless failure also occurs at the fourth stage that is failure to correctly solve the task. She also added that if the learner cannot explain the meaning of the question in his or her own words but is able to perform the task correctly, it is likely that comprehension has occurred even though it was not verbalised (Newman, 1977). Sometimes learners may not be able to explain the meaning of word problems in their own words due to other factors such as problems with speech, shyness even when they understand what the question requires them to do. If they can show understanding in the way they answer the question by showing the processes needed to gain a correct solution and they get it, then it is assumed that they have abilities in all strategies.

The “Performance strategies” or “Criterion for Error causes” by Newman are consistent with the four basic principles of problem solving according to Polya (1945) which are: Understanding the problem, devise a plan, carryout the plan and look back. It can be assumed that by understanding the problem, Polya implied that learners must be able to read the problem and state in their own words what the problem requires them to do. Devising a plan could involve selecting the processes needed to solve the problem, while carrying out the plans involves performing the process skills to gain an answer and looking back includes checking if the processes and the encoding of the final answers are correctly done. Under appropriate instructional conditions, word problems not only can be negotiated successfully by the learners, but they can also be the vehicle to promote greater mathematical reasoning and achievement (Manroe, Panchyshyn and Bahr, 2006). Therefore, teachers should expose learners to reading in mathematics so that they can make predictions, monitor understanding, determine importance and make connections (Halladay and Neumann, 2012).

The above argument informs this study on the criterion for error causes or performance strategies which can be used for error identification in algebraic word problems. It was therefore worth investigating if teachers in their teaching of algebraic word problems to grade eights do apply Newman’s performance strategies to enhance learners’ understanding of the topic.

2.4 Learners’ errors in algebra

Learners make a lot of mistakes when solving algebra, especially algebraic word problems. Studies show that there is a relationship between learners’ poor performance in mathematics and the errors they make when solving algebra (Norton, 2009; Luneta and Makonye, 2010, Mukuka and Shumba, 2016). Fuchs, Fuchs, and Compton (2012) were of the notion that learners’ poor performance could result mainly due to errors with mathematical concepts. Particularly, these errors occur when learners have to solve algebraic word problems. The aspect of creating equations to represent the relationship between the quantities also remains a challenge for most learners (Kieren, 2007). Drijvers, Goddijn, and Kindt, (2011) argued that learners find it difficult to transfer from arithmetic to algebra and this could be the reason for the errors they make in solving algebra. Nalube (2014) observed that conjoining error was common among

learners as they often replace a variable by any number for them to add and subtract algebraic expressions easily. Maat, Ibrahim and Zakaria (2010) also revealed that errors in solving equations such as algebraic word problems are due to lack of understanding the syntax and structure of algebra. Lack of conceptual, procedural and strategic knowledge by learners was one of the challenges faced in answering algebraic linear equations (Koji, Mulenga and Mukuka, 2016). Lambdin (2009) stated that learners who have learnt mathematics without understanding are often successful only in solving problems similar to those they have already seen, and are unable to see how mathematical ideas are related or useful. These learners often see the subject itself in a negative light, viewing it as arbitrary and mysterious – a subject which only geniuses can master.

Some researchers used 'Newman's Error Analysis Model' (NEA Model) in learners' error analysis and established that in word problems learners made errors on the stages of reading and comprehension as the two were related to language (Mukunthan, 2013; Salihu, 2017; Santoso, Farid and Ulum, 2017). Scholars also revealed that the reasons for learners' challenges and the errors they make in Mathematics are linked more to their poor language proficiency than to their lack of computation ability (Ellerton, 1989; Oviedo, 2005; Fuchs, Fuchs and Compton, 2012; Le Febvre et al, 2010; Egodawatte, 2009; Riordain and O'Donoghue, 2009; Seng, 2010; Wright, 2014; Ural 2016; Boonen, Koning and Jolles, 2016). This means learners' computation skills challenges are due to their inability to identify and understand key words in the problem statement. Montague (2005) indicated that key words play an important role in helping problem solvers to explain the meaning of word problems in their own way. Emphasising or highlighting key information is designed to help learners restate the mathematical problems in their own words, thereby strengthening their understanding of word problems (Cunningham and MacGregor, 2014; Polya, 1957).

From the above studies, it can be noted that learners make a lot of errors in word problems due to a number of factors such as language problems, reading abilities, conjoining and failure to understand key issues in the problem. These errors could hinder their understanding of further mathematics which needs them to have knowledge of algebra, as it has been noted that algebra is used in many topics in mathematics, directly or indirectly, hence its importance cannot be overemphasised

(Musonda, 2017). Therefore, this helped me to focus my study on teachers' classroom practices in order to see how they probe learners' abilities in reading, comprehension, transformation, as well as process skills and encoding in their classrooms when teaching algebraic word problems. That also made me curious to observe how teachers identified and corrected learners' errors at each of these stages when solving algebraic word problems. It is vital that teachers probe learners' abilities at every stage of solving algebraic word problems in order to lessen the errors. It is imperative that error analysis is done to enable effective learning. Logically a more focused research on how teachers identify and address learners' errors can help to correct errors and lessen the challenges Zambian learners face in mathematics and ultimately improve the results in the subject.

2.5 Teachers' classroom practices and learners' errors

It is imperative that teachers' classroom practices aim at identifying and addressing learners' errors especially in word problems such as algebraic word problems, where scholars have indicated that learners do have challenges. Aguele, Ojugo and Imhanlahimi (2010), revealed that the direct instruction was a more effective strategy for the remediation of process errors committed by learners in mathematics. They recommended that enough practice activities should be given to learners during class sessions to assist them develop mastery of content taught. Borasi (1987) also observed that errors can be a powerful tool for diagnosis of learning difficulties and consequently directing remediation. Therefore, teachers need to establish what learners can do unassisted and what they can do by collaborating with more capable peers or being assisted by more knowledgeable others, in this way they can provide the necessary mediation (Vygotsky, 1978).

In addition, Suh (2007) looked at what classroom practices would promote mathematical proficiency for his learners. Some of his learners loved the challenges he presented in mathematics, while others already showed signs of mathematics avoidance and even mathematics phobia. It was so clear to him that these five intertwined strands, namely conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition were the critical components for developing mathematically gifted learners. Moreover, Bush (2011) revealed common algebra misconceptions and trends that could help guide instruction

for middle school mathematics teachers. The scholar established that the findings had direct implications for classroom practice and further confirmed the need for strong and knowledgeable teachers of mathematics at the elementary and middle grades. This is crucial because at those levels learners need to build a strong foundation in mathematics, hence the need for knowledgeable teachers. Sammons (2018) added that for all learners, including those who are attempting to put their ideas down on paper for the first time, those who are refining their general writing skills, and those who are learning to express their mathematical ideas in writing, some of the productive learning experiences result from observing teachers as they model the writing process and speak aloud about their thinking as they do. O'Connel and Croskey (2018, p.60) added that, "Writing is a process through which we recognise our ideas, develop conjectures, and gain insights. It is not content itself, but a means of exploring and expressing content." Teachers need to treat learners as active participants in their own learning so that they are helped to redefine their situation definitions in order for them to make the transition from their respective actual developmental levels to potential developmental levels in the ZPD (Wertsch, 1979). The third principle of social constructivism derives from the sociology of knowledge and acknowledges that reality is constructed intersubjectively, that is it is socially negotiated between significant others who are able to share meanings and social perspectives of a common lifeworld (Berger and Luckman, 1966). Hence, the importance of the teacher who is a significant other to the learners.

In addition, Osterholm (2006) who in his study compared reading comprehension of three different texts: two mathematical texts and one historical text among 95 upper secondary and university students, and results revealed that although the university students had studied more mathematics courses than the upper secondary learners, there was only a small and insignificant difference between these groups regarding reading, comprehension of mathematical text with symbols. The findings suggested that there was a need for more explicit teaching of reading comprehension for texts including symbols. Sociocultural theory stresses that the most beneficial learning environment is one where a dynamic interaction between teachers, learners and tasks provides an opportunity for learners to create their own understanding during interaction with others (Atkinson, Derry, Renkl and Wortham, 2000). Atherton (2005) emphasised that in a sociocultural classroom, learners are active creators of meanings

and the role of the teacher is to guide learners to gain meaningful understanding of the learning material. However, Crespo (2000) argued that, knowing what to look for and what to do with learners' mathematical utterances, whether in written or in oral form is not a trivial task for any teacher. Brodie (2014) also stressed the importance of teachers paying attention to learners' errors so that effective teaching can be done.

Furthermore, Chauraya and Brodie (2013) indicated that, research on learner errors in mathematics education was beginning to focus on how teachers can learn to identify and engage with the reasoning behind those errors. Nalube (2011) asked student teachers to explain their reasoning of learner responses to tasks designed around algebraic error and comment on how they would deal with them. The findings revealed that particular attention was required in the programme on how errors were thought about in mathematics and the kinds of opportunities that student teachers needed to engage in, in order to build up their knowledge and reasoning about learner errors. Even and Tirosh (2002) Encouraged a classroom culture where learners are allowed to make conjectures, explain their reasoning, validate their assertions, discuss and question their thinking and the thinking of others, and argue about what is mathematically true. This is practically important as it is a way of developing learners' competences.

This shows how important it is for the teachers to identify and address learners' errors in mathematics, as it leads to conceptual understanding. Improving student learning is the primary goal of every teacher of algebra. We seek strategies to help all learners learn important algebraic content and develop mathematical practices. Wertsch (1985, p.67) argued that, "ZPD is to deal with two practical problems in the learning situation: the assessment of learners' intellectual abilities and the evaluation of instructional practices." In addition, incorporating these ideas into teacher education programs can better prepare prospective teachers for their initial classroom experiences, which often are in algebra settings" (Zbiek and Larson, 2015, p.697).

Many other scholars have talked about good classroom practices to consider when solving algebraic problems in mathematics with regards to learners' errors (Travis, 1981; Herholdt and Sapire, 2014). For example Holohan (2018) discouraged teachers from jumping in too quickly to help or assume learners know how to tackle the problem

but to allow them explore the problems on their own first. As learners continue to practice, they can do certain tasks independently in activities which were previously executed with assistance. The changes the learners gain in understanding help them to find a way of attempting the problems which they were unable to solve even with assistance (Siyepu, 2013). Brodie (2000) stated that teaching in the ZPD requires teachers to teach through questioning and prompting of learners' ideas. Teachers also need to show learners the correct use and presentation of tools through the way they speak and write when solving mathematical problems (Wertsch, 1990).

What comes out clearly in the above literature is the role of the teachers in learners' errors. It is clear from the above literature that teachers must endeavour to identify and address learners' errors in their daily classroom practices, if learners are to become competent in solving word problems. Teachers must also engage learners in their own learning. Hence to adhere to this issue, this study explored how teachers teach algebraic word problems clearly observing if they employ deliberate measures such as engaging learners in reading algebraic word problems, probing their understanding, and paying attention to learners' process skills and encoding abilities to reveal and address learners' errors.

2.6 Studies on learners' errors in algebra and related topics, and gaps identified

A study conducted in Australia to analyse sixth grade learners' errors on written mathematical tasks provided a great insight to this study. The study was done by Newman (1977), on 124 low achieving learners drawn from a total of 917 sixth grade leaners in 31 classes of 19 different schools in the western suburbs of Melbourne, Victoria. Learners were given a test to write and also interviewed. Results indicated that the 124 low achievers who were interviewed made 3002 errors when they first attempted to solve the items on the mathematical tasks test. When these errors were analysed and classified according to the criterion for error causes, the following results were obtained: 13 percent of the errors were because learners were unable to read the questions accurately enough for them to understand the meaning of tasks; 22 percent of the errors were because learners were unable to explain the meanings of the questions in their own words; 35 percent of the errors were because learners failed to apply relevant knowledge, skills or principles; 12 percent of errors arose with learners

who were able to read and understand the meaning of the tasks but were unable to indicate that they had identified the required mathematical processes needed to obtain correct solutions; 25 percent of initial errors were due to other reasons such as carelessness, lack of motivation or task form.

The findings of Newman's study are very important because they informed this study that learners do make errors in the process of solving mathematical tasks. Moreover, the performance strategies of Newman's (1977) study helped me to formulate research objectives and research questions. However, Newman's study was centred on analysing grade six learners' errors of written mathematical tasks. This research left a gap since it was centred on learners. Therefore, there was need for another study to look at teachers' classroom practices in addressing learners' errors in algebraic word problems.

Haghverdi (2012) conducted a study to recognise learners' difficulties in solving mathematical word problems from the viewpoint of teachers. 52 mathematics teachers were chosen randomly to participate in the study. The results showed that the learners' difficulties were randomly sprung from their disabilities in representation and understanding of word problems, making a plan and defining the related vocabularies. The findings also revealed that, the causes of learner difficulties were text difficulties, unfamiliar contexts in problems and using inappropriate strategies. Moreover, the results from content analysis demonstrated that almost half of teachers' responses (51%) indicated that solving mathematical word problems was difficult for learners because they struggle with representation and understanding the problem. Two other difficulties cited in teachers' responses involved learners' inability to make a plan to solve the problem (31%) and a lack of vocabulary knowledge (10%).

This study by Haghverdi (2012) helped me to focus on studying teachers' classroom practices in addressing learners' errors, knowing very well that teachers are aware of learners' difficulties in word problems. However, Haghvedi's (2012) study focused on studying learners' difficulties whilst this study's focus is on teachers' classroom practices in addressing learners' errors.

Another study which was conducted in Kenya by Mulungye (2010) examined the various errors and misconceptions committed by learners in algebra with the view to

exposing the nature and origin of the errors and misconceptions in secondary schools in Machakos district. Teachers' knowledge of learners' errors was investigated together with strategies for dealing with them. The study used a sample of four hundred and thirty form two learners and fifteen mathematics teachers of the respective classes at the time of the study. Data comprised of the results from mathematics learners' tests, learner interview schedule, and mathematics teachers' questionnaires. The findings revealed that most (63%) learners experienced difficulties with the word problems while equations had the least percentage (22.3%) of errors. The results also revealed that teachers were aware of the errors that learners make.

The importance of Mulungye's (2010) study is that it helped me to focus on algebraic word problems since most of the errors were committed by learners in word problems as per study findings. It also helped me to focus on teachers' classroom practices in addressing learners' errors due to the fact that the study established that teachers were aware of errors that learners make.

Zakaria and Maat (2010) conducted a study in Indonesia whose purpose was to determine learners' errors in learning quadratic equations. A sample of 30 learners from a secondary school was selected. A diagnostic test which included three components: factorisation, completing the square and quadratic formula, was used as the instrument of this study. The type of error was based on Newman Error Hierarchy Model that includes reading type error, comprehension, transformation, process skill, and encoding error. The findings showed that most learners made errors in transformation and process skill in solving quadratic equations. There was no error found in reading. Zakaria and Maat (2010) attributed learners' errors in solving quadratic equations to weaknesses in mastering topics such as algebra, fractions, negative numbers and algebraic expressions.

This study is important to my study as it helped me to focus on algebraic word problems by the fact that there was a perceived weakness in learners mastering algebra. The study also helped me to focus on exploring teachers' classroom practices using the performance strategies of Newman Error Hierarchy Model. However, Zakaria and Maat (2010) study explored learners' errors in learning quadratic equations while my

study focused on exploring teachers' classroom practices in addressing learners' errors in algebraic word problems.

Moreover, Wright (2014) investigated factors affecting learner performance in Algebraic Word Problems solutions. The study was designed to investigate the potential significance of several learner algebraic word problem solving skills. Findings revealed that statistically significant rank-order correlations were found in comparisons of participant performance in: ability to identify algebraic operation; ability to recognise relational statements, and; ability to translate text into equations when compared with correctly solved algebraic word problems scores. Statistically significant differences in mean number of correctly solved algebraic word problems were found between grade levels 9, 10, and 11 and also between the following courses: Algebra I, Algebra II, and Pre-calculus.

Wright's (2014) study also helped me to focus on algebraic word problems, since the study established that leaners made errors regarding operations when solving algebraic word problems across all grade levels sampled. The recommendation was that there should be increased emphasis on the teaching and practice of identifying operations as they are referenced in algebraic word problem statements at all grade levels where learners encounter such tasks. I became interested to explore what practices teachers employ if such errors arise when teaching algebraic word problems. However, this study was different in the sense that it focused on investigating the potential significance of learners' algebraic word problem solving skills across different grade levels. The study employed quantitative research design. The current study, however, focused on exploring teachers' classroom practices in addressing learners' errors in algebraic word problems at grade eight level using a qualitative research approach.

Adu, et al (2015) also examined errors learners made in solving linear equation word problems with a view to expose the nature of these errors and make suggestions for classroom teaching. A diagnostic test comprising 10 linear equation word problems was administered to a sample (n=130) of senior high school first year Home Economics and General Arts learners in a senior high school in Central Region of Ghana. The errors learners made were identified based on the modified Newman Error

Hierarchical Levels (NEAL), which comprise reading, comprehension, transformation, process skills, and encoding errors.

The results revealed that majority (60%) of the learners attempted most of the questions with a few (2%) arriving at the correct answer, which implied that learners had challenges in tackling linear equation word problems. The study further revealed that about 75% of the learners made comprehension errors; 86% made transformation errors which occurred during the translation of the statement to algebraic form; 84% made process skills errors which occurred during computational process, and finally 86% made encoding errors which occurred at the final stage of the work. The study concluded that, leaners' errors in solving linear equation word problems were due largely to their inability to comprehend and interpret the sentences in order to proceed with the process and encoding skills.

The importance of this study lies in the fact that it helped me to think of exploring how teachers engage learners in reading, understanding (comprehension and transformation), and how they enhance learners' process skills and encoding abilities in algebraic word problems to ensure that learners comprehend the concepts so that their errors noted in those stages are lessened. However, the difference is that Adu, Assuah, and Addo (2015) in their study examined learners' errors in solving linear equation word problems whereas this study focused on teachers' classroom practices in addressing learners' errors in algebraic word problems.

Furthermore, Mukunthan (2013) conducted a study on learners' errors on word problems whose objectives were: to identify the reasons for learners' weaknesses in word problems; to find the types of errors made when solving word problems by boys and girls; and to find the types of errors made when solving word problems by learners of different grade levels. He used a descriptive study and data collection in the study was mainly based on several learners in two grade levels (Grade 4 and 5) and at least a single measurement of performance for each individual within each grade level.

A question paper containing 5 word problems covering all four mathematical operations was used to collect data. Results revealed that learners made errors across all five questions as follows: 40.9% reading, 31.8% comprehension, 13.6% transformation, 9.1% process, and 4.5% encoding. The study established that learners'

performance in word problems was very weak. Thus the study recommended that teachers have to identify error patterns in learners and take remedial measures.

This study helped me to focus on word problems, particularly algebraic word problems since it was revealed that learners' performance in word problems was very weak. Mukunthan's (2013) study also helped me to consider focusing on the teacher in order to see what they do in their classrooms to identify and correct learners' errors in reading, comprehension, transformation, process, and encoding. However, Mukunthan's (2013) study is different from the current study since it focused on learners' errors in word problems in general, while the current study focuses on the teacher regarding learners' errors identification and correction in algebraic word problems.

Ural (2016) examined 7^{th} grade learners' mistakes in algebraic expressions. The aim of the study was to determine learners' achievement rates in algebraic expressions and mistakes they made. Hundred 7^{th} grade learners from four different schools participated in the study. The study used a qualitative research which was carried out using a survey design, and data was analysed using descriptive analysis. A test with six open-ended questions about algebraic expressions was used as the measurement tool.

Results revealed that learners' achievement rate was 68% in addition operations in the algebraic expressions, 43% in expressing a geometric representation as an algebraic expression. The overall average was 69%. It was determined that the main reasons were failure to attribute a meaning to the unknown and therefore doing the operations by assigning a value to the unknown.

Salihu (2017) investigated grade 11 learners' errors when solving algebraic word problems in South Africa. The method employed was a sequential explanatory mixed approach. The quantitative part involved 150 learners while qualitative analysis was used by conducting the Newman (1977) interview format with eight learners to find out areas where errors are made and what kind of errors they are. Findings indicated that 90 learners demonstrated unfitness due to poor linguistic proficiency, while the remaining 60 learners fall into three main categories, namely those who benefited from

research unpacking meaning; those who lacked transition skills from arithmetic to algebra; and those who lacked comprehension and calculation knowledge.

The findings of Salihu's (2017) research helped me to give direction to the current study by focusing on exploring how teachers engage learners in reading and explaining in their classrooms in order to identify and correct learners' errors as that can lead to proficiency in both language of mathematics, and common language, as well as helping learners to find it easy to translate from arithmetic to algebra.

A study within Zambia investigated secondary school learners' errors and misconceptions in algebra among grade 11 learners in two selected schools in Petauke district in Eastern Province of Zambia. This study was done by Mbewe (2013) from the University of Zambia. The study was done with a view to exposing the nature and origin of these errors and make suggestions for classroom teaching. The main finding of this study was that the misconceptions which leaners experience in algebra were attributed to lack of conceptual knowledge and understanding. Hence, the study recommended that teachers and learners should openly talk about misconceptions in the classroom during the teaching and learning process. The study further recommended that individual attention to learners should be given by the teacher in order to understand learners' mathematical reasoning.

This study by Mbewe (2013) helped me to focus on algebra and also to explore classroom teaching in order to see how teachers pay attention to individual learners when teaching algebraic word problems for them to ensure that learners acquire conceptual understanding. I also became particularly interested in finding out if teachers and learners openly talk about misconceptions and errors in the classroom during teaching and learning of algebraic word problems as per recommendation of the study. Mbewe's (2013) study is different from the current study in the sense that it explored learners' errors and misconceptions in algebra at grade 11 level, while the current study focuses on teachers' practices in addressing learners' errors in algebraic word problems at grade eight level.

Another study in Zambia was done by Musonda (2017) who examined the relationship between algebraic competences and emotional intelligence of first year Bachelor of Science in Mathematics and Science education students at Copperbelt University. All

the 143 first year students in that course in 2016 were purposively selected for the study. Emotional intelligence was measured using the Schutte Self-Report Emotional Intelligence Test (SSEIT), and algebra competences were measured through a grade 12 level algebra achievement test and a University level algebra achievement test constructed by the researcher. Results indicated that there was a significant positive correlation between Grade 12 level and first year University level algebraic competences. Results also showed that there was no significant relationship between students' algebraic competencies and their emotional intelligence.

This study by Musonda (2017) helped me to focus on algebra as a topic since the scholar in his study highlighted the importance of algebra, and established that there was a strong correlation between grade 12 level Algebraic competences and the first year university level Algebraic competences. This to me implied that algebra must be well taught in secondary schools so that learners can gain competences which can later be applied in their university education. Hence my interest in exploring teachers' classroom practices in addressing learners' errors in algebraic word problems. However, Musonda (2017) examined the relationship between algebraic competences and emotional intelligence of first year students, and used a quantitative method, while the current study explored teachers' classroom practices in addressing learners' errors in algebraic word problems at grade eight level and used a qualitative approach.

Moreover, Mukuka and Shumba (2016) evaluated the perceptions of 73 student teachers at Mukuba University on the nature and purpose of algebraic proofs. The students were presented with a mathematical reasoning and proof arguments assessment tool followed by a 10 member focus group interviews. Algebraic tasks were presented to these students. These tasks included: proof construction on simple number and set theory; assessment of given mathematical statements and arguments to validate them, and; validating formally structured inductive proofs. Findings revealed that more than 75% of the student teachers had limited understanding of the nature and purpose of algebraic proofs and they believed that proof construction and validation entailed inspection of a few examples and the testing of single extreme cases. Moreover, the study also established that concrete examples of empirical forms of mathematical arguments which student teachers perceived as valid were found but these demonstrated limited forms of mathematical reasoning and comprehension.

This study is important as it helped me to consider exploring teachers' classroom practices in algebra, particularly algebraic word problems to see how teachers engage learners in learning so that conceptual understanding occurs. The fact that many student teachers had limited understanding of the nature and purpose of algebraic proofs motivated me to focus on algebra and particularly how teachers' classroom practices reveal and address learners' errors when teaching algebraic word problems.

Moreover, Luneta and Makonye (2010) investigated learners' errors and misconceptions in calculus and the results revealed that the errors and misconceptions in Calculus were due to knowledge gaps in algebra. However, some errors learners made were as a result of other psychological factors such as carelessness, anxiety, overconfidence, and lack of motivation (Newman, 1977; Egodawatte, 2009). This study helped me to focus on algebraic word problems since the scholars observed that the knowledge of algebra is cardinal in understanding other topics such as calculus.

Having looked at studies on learners' errors in algebra and related topics, and gaps identified, below is a summary (Table 1) of literature review showing themes and key issues as well as how they are related to, and informed the current study.

2.7 Summary of the chapter

This chapter presented a number of issues arising from literature review on the: importance of algebra; criterion for error causes or performance strategies; learners' errors in algebra; teachers' classroom practices and learners' errors; as well as studies on learners' errors in algebra and related topics, and gaps identified. In all the studies highlighted in this chapter, what comes out is that a number of studies focused on the analysis of learners' errors rather than on the teachers' practices and their influence on the errors that learners make. A number of reasons have been outlined as to why learners make those errors and recommendations have been made in terms of teachers' practices to lessen those errors. Therefore, that was seen as a gap, hence, this study explored teachers' classroom practices in order to see how they help in identifying and addressing learners' errors in algebraic word problems using the criterion for error causes or performance strategies as outlined by Newman (1977). The themes and key issues in this chapter are as summarised in Table 1.

Table 1: *Summary of themes and key issues related to the study*

Themes	Key issues	Relating to the current study
Importance of algebra	• Requirement for STEM education. • Pre-requisite to many topics and subjects. • Application in real life. • Gatekeeper. • Language of generalisation • Order of universe.	• My study has been informed on the importance of algebra in real life. • Understanding the importance of algebra in life led to my focus on algebra and identification of the gap in the context of my study.
Criterion for error causes or performance strategies	• Reading ability • Comprehension ability • Transformation ability • Process skills • Encoding ability (Newman, 1977)	• My study has been informed on the performance strategies to use when solving or researching on word problems. • Performance strategies helped me to formulate research objectives and questions of the study, and also helped me with data analysis.

Learners' errors in algebra	• Learners make a lot of errors. • Difficult to transfer from arithmetic to algebra. • Conjoining errors in algebra. • Lack of understanding. • Learners make errors on stages: Reading, comprehension, transformation, process, and encoding. • Poor language proficiency. • Lack of computational abilities. • Inability to identify and understand key words.	• My study was informed that learners make errors in algebra mainly due to the following reasons: (i) Failure to read the questions correctly. (Failure to recognise words and symbols). (ii) Failure to understand the meaning of the questions. (Difficult in understanding the structure and syntax of algebraic word problems). • Learners' errors in algebra and reasons for those errors helped me to focus on exploring teachers' classroom practices in addressing learners' errors in algebraic word problems to ensure that learners acquire conceptual understanding.
Teachers' classroom practices and learners' errors.	• Teachers' classroom practices should aim at identifying learners' errors. • Effective strategies and remediation. • Enough practice for learners during class. • Errors as powerful tool for diagnosis of learners' difficulties and directing remediation. • Teachers need to be aware of learners' errors. • Allow learners to explore problems on their own first. • Openly talk about misconceptions. • Provide individual attention to establish individual reasoning.	• The current study was informed on the importance of learners' errors, as they show that learners do reason and that teachers' practices should be directed towards identifying and correction of learners' errors. • The understanding of teachers' classroom practices and learners' errors helped me to focus on exploring teachers' practices with regards to the identification and correction of learners' errors.

Studies on learners' errors in algebra, related topics, and gaps identified	• Analysis of learners' errors in written mathematical tasks. • Recognising learners' difficulties in solving mathematical word problems. • Determining learners' errors in quadratic equations using performance strategies. • Examining various errors and misconceptions by learners' in algebra. • Teachers and learners to openly talk about misconceptions during teaching and learning • Teachers to provide individual attention to learners when teaching • Identifying reasons for learners' errors in word problems.	• My study was informed on the focus of many studies reviewed. It was established that many studies focussed on looking at learners' errors and offering suggestions for classroom teaching. Hence, I became interested in establishing whether the suggested practices form part of teachers' practices when teaching algebraic word problems. • These studies helped me to identify the gap, hence I focussed on exploring teachers' classroom practices in light of learners' errors in algebraic word problems

CHAPTER 3

RESEARCH METHODOLOGY

3.1 Introduction

This chapter discusses the general methodology employed in the study. Methodology involves describing the strategies that were used in carrying out the study. It endeavours to highlight the research paradigm, research design and approach, population, sampling techniques and study sample, data collection methods, data collection instruments, data analysis, rigor and ethical considerations. According to Kazdin (1992), research methodology refers to the principles, procedures, and practices that govern the research.

3.2 Research paradigm

Research paradigms are preferred ways of understanding reality, building knowledge, and gathering information about the world (Tracy, 2013). This study employed a constructivist or interpretive paradigm (Creswell, 2003). Constructivists view knowledge as socially constructed through language and interaction, and reality as connected and known through society's cultural and ideological categories. The Constructivist paradigm emphasises the importance of examining the world from participants' point of view as they believe that both knowledge and reality are constructed and reproduced through communication, interaction, and practice (Tracy, 2013). Hence, this paradigm was appropriate as I was interested in exploring teachers' classroom practices naturally in their classrooms where teaching is done through communication, interaction and practice.

3.3 Research design

Research designs according to Creswell (2009, p.4), "are plans and the procedures for research that span the decisions from broad assumptions to detailed methods of data collection and analysis." This study employed a descriptive case study design. A case being mathematics teachers in Isoka district. Berg (2007) described a descriptive case study as a method of collecting information by interviewing or observing a sample of individuals. It can also be used to collect information about people's attitudes, tone of

voice, gestures, opinions or habits on any educational or social issue. I employed a descriptive case study design because I wanted to bring out issues as they exist in the natural setting. This enabled me to report on the findings regarding the topic, teachers' classroom practices in addressing learners' errors in algebraic word problems. This was through the use of lesson observations, semi-structured interviews, document analysis and FGDs.

This particular study employed a qualitative approach. This was so because the study sought to ask questions that were not based on no or yes as answers. This was done in order to allow the teachers and learners to explain their thoughts, opinions, ideas or beliefs in a detailed manner. Dooley (1984) noted that, when research is based on qualitative methodology, it makes use of non-quantitative observations which are made in the field and analysed in a non-statistical way. Seale (2004) added that qualitative approach involves intensive studying of a single setting or a number of people. In addition, qualitative approach involves the researcher interacting, observing and interpreting meaning from the cases being studied. It also involves the researcher describing kinds of characteristics of people and events without comparing them in terms of measurements and amounts or numbers. Finally, in this regard, qualitative research reminds us that reality and viewpoints are not fixed variables in life. This approach was appropriate for this study because I was interested in studying teachers' classroom practices by interacting with them through lesson observations and interviews in their natural classrooms. After describing the research design and approach for my current study, in the next section I describe the place where this study was carried out and the reasons for choosing the place.

3.4 Study area

The study was conducted in Isoka district (Figure 3) of Muchinga province of Zambia particularly at three government secondary schools. There are many motivating factors that could influence the researchers' choice of the study site, such as; the nature and incidence of the problem, research time frame, and data accessibility, clients' interest and instructions, resource availability, performance in particular field, goals and objectives of the study (Msabila and Nalaila, 2013). Therefore, I selected Isoka district as the study area for my study because the performance in mathematics in Muchinga

Province from 2012 to 2017 has been below 40% and Isoka being one of the nine districts in the province is a major contributor to the statistics. This reason is backed by the Examinations Council of Zambia 2016 general performance analysis which indicated that Muchinga province average scores in mathematics for 2012, 2014 and 2016 were 34.56%, 35.31% and 36.05% respectively (ECZ, 2016, p.59).

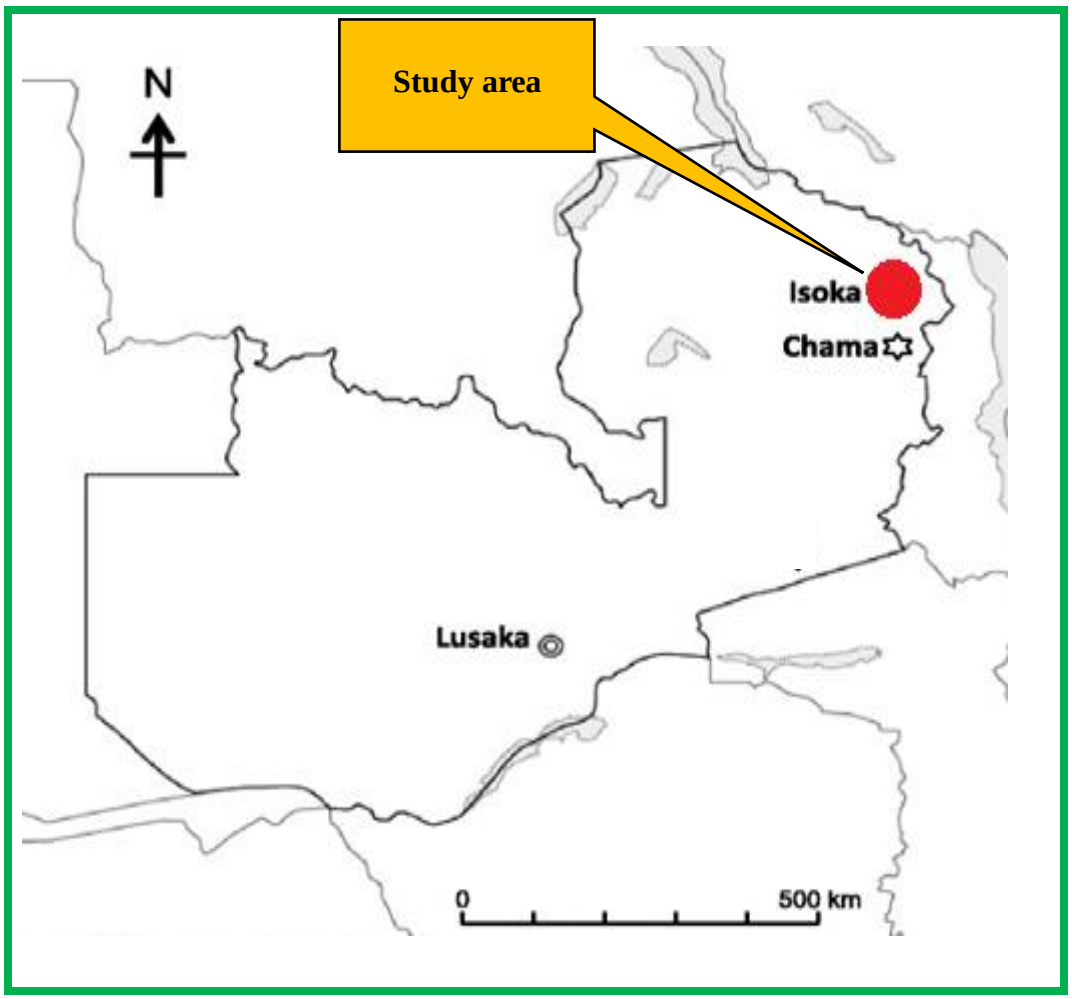

Figure 3: *Map of Zambia showing location of Isoka district of Muchinga province*

3.5 Study population

"The population is all individuals of interest to the researcher," (Marczyk, DeMatteo and Festinger, 2005, p33). Therefore, the individuals of interest to me were teachers teaching grade eights at that time, and grade eight learners from the three secondary schools in Isoka district of Muchinga Province, Zambia. The choice of teachers was based on the reason that teachers are key regarding identification and correction of learners' errors when teaching algebraic word problems. In addition, I included grade

eight learners to help me triangulate data regarding teachers' classroom practices in addressing learners' errors in algebraic word problems.

3.6 Study sample

Researchers typically study a subset of the population, and that subset is called a sample. Because researchers may not be able to study the entire population of interest, it is important that the sample be representative of the population from which it was selected (Marczyk, DeMatteo and Festinger, 2005). Neumann (2014, p.246) stated that, "when we sample, we select some cases to examine in detail, and then we use what we learn from them to understand a much larger set of cases." Therefore, in this study the sample size I employed comprised of five (2 from school A, 2 from school B & 1 from school C) grade eight teachers of mathematics and twenty-five (10 from school A, 10 from school B & 5 from school C) grade eight learners of algebraic word problems from three secondary schools (school A, school B & school C), making a total sample of 30 participants in Isoka district of Zambia. This was a reasonable sample size for a qualitative study because it provided data needed to fulfil the requirements of efficiency, representativeness, reliability and flexibility (Merriam, 1998). Having provided the study sample for this study, I now discuss the sampling techniques in the next section.

3.7 Sampling techniques

In this study, I employed purposive sampling technique which is homogenous in nature. Purposive sampling is appropriate to select unique cases that are especially informative as it uses the judgement of an expert in selecting cases with specific purpose in mind (Kothari, 2004; Neumann, 2014). This is because the study was a naturalistic investigation which has design strategies, flexibility and purposeful sampling (Best and Kahn, 2009). Purposeful sampling is powerful in selecting information rich in cases for in-depth analysis (Merriam, 1998). Therefore, in order to ensure that grade eight teachers take part in this study, purposive sampling technique was employed (Maxwell, 2005). I purposively selected grade eight teachers of mathematics who were teaching algebraic word problems at that time. The grade eight learners in the five classes for these teachers (Grade 8V & grade 8W from school A,

grade 8X & grade 8Y from school B, and grade 8Z from school C) automatically became learner participants.

Five grade eight learners from each class (Grade 8V, 8W, 8X, 8Y and 8Z, from the three schools) were purposively selected (It is important to note here that the classes have been written using pseudonyms). These learners were purposively selected with the help of the teachers because I aimed at involving three categories of grade eight learners (High performers, middle performers and low performers) in mathematics and algebra in particular. The criteria of choosing the high performers (2 from school A, 2 from school B, and 1 from school C), middle performers (4 from school A, 4 from school B, and 2 from school C) and low performers (4 from school A, 4 from school B, and 2 from school C) was based on their general performance in mathematics. I did this in order to establish how teachers identify and correct learners' errors in all these categories of learners when teaching algebraic word problems, and to see what they do to enhance these learners' understanding of algebraic word problems after establishing their errors. If I had not considered the abilities of learners in sampling, it would have distorted the establishment of the learners' views on teachers' classroom practices in addressing learners' errors and their challenges in algebraic word problems. The challenges of these categories of learners could be idiosyncratic and so needed to be explored. Purposive sampling, homogenous in nature was used because I wanted to seek answers to questions by examining various social settings with the grade eight teachers who exhibit these settings and share similar characteristics (Patton, 2002; Suri, 2011; Etikan, Musa, and Alkassin, 2016). Having outlined the sampling techniques for this study, I now turn to the next section where I discuss data collection methods and instruments.

3.8 Data collection methods and instruments

In this study, data was collected using lesson observations protocols, semi-structured interview guides, document analysis schedules and FGDs guides (Denscombe, 2003). A tape recorder was also used when conducting semi-structured interviews and during FGDs. According to Kothari (2004), under observation method, the information is sought by way of investigator's own direct observation without asking from the respondent. The interview method of collecting data involves presentation of oral-

verbal stimuli and reply in terms of oral-verbal responses. Dawson (2002) advised that when using a tape recorder, it is important to hear your own voice as well as that of the interviewee so that you know what answers have been given to which questions.

For this study, four research methods were used to collect data from the respondents, and these included: Lesson observations, Semi-structured interviews, Document analysis, and FGDs. The following subsections provide more details about data collection methods and procedures regarding the same.

3.8.1 Lesson observations

In this study, I used lesson observation protocols to collect data as the teachers were teaching algebraic word problems to the grade eights. The rationale for lesson observations was to see how teachers engaged grade eight learners in reading, how they probed learners' understanding and what they did to enhance learners' process skills and encoding abilities when teaching algebraic word problems (see Appendix C for details, p.120). According to McMillan and Schumacher (1993), lesson observation is a tool which provides information about the actual behaviour. Creswell (2014), added that, direct observation is useful because some behaviour involves habitual routines of which people are hardly aware. Patton (2002) also observed that, observation is a data collection procedure whereby researchers try to understand what is happening in a given setting by paying attention, video recording, watching and listening carefully.

Hence in this study, I used video recording to capture lesson observations in totality. According to Merriam (1998), one of the advantages of using a video camera to record field observations is that it allows the researcher not only to capture the physical environment but also to revisit the images later and recall the experiences during analysis.

3.8.2 Semi-structured interviews

Semi-structured interviews were conducted with the teachers after lesson observations in algebraic word problems. The rationale of the interviews with the teachers was to have an in-depth understanding, opinions, and views pertaining to their classroom

practices in identification and correction of learners' errors in algebraic word problems. I used semi-structured interview guides for this purpose (see Appendix D for details, p.123). According to Kvale (1996), researchers across the sphere contended that interviews are the best way to collect data because it helps the researcher to establish feelings, opinions, attitudes, views, and are useful in gathering in-depth data.

3.8.3 Focus group discussions

In this study, I used FGDs in order to collect data from grade eight learners on teachers' classroom practices in algebraic word problems. This was purely meant for the purpose of triangulation, in order to see if what I observed during lesson observations and semi-structured interviews with teachers was consistent with what the learners said. Cohen, Manion and Morrison (2007) described triangulation as the use of two or more methods of data collection in the study of some aspects of human behaviour. For this purpose, I used FGD guides (see Appendix F for details, p.127). FGDs with five learners each were held after lesson observations and semi-structured interviews were conducted with the teachers. In order to enhance trustworthiness of these qualitative results, I audio recorded the proceedings of FDGs and this also helped me to capture data in totality.

3.8.4 Document analysis

In this study, I also used document analysis method by looking at the lesson plans for the teachers who participated in the study in order to find out if these documents reflected how the teachers intended to engage grade eight learners when teaching algebraic word problems regarding learners' errors identification and correction. For this purpose, I used document analysis protocol (see Appendix E for details, p.124). I now discuss data collection procedures and time line in the next section.

3.9 Data collection procedures and time line

The actual data was collected in the first term of the school calendar for 2019. For school A data was collected between 6th March, 2019 and 27th March, for school B between 10th March, 2019 and 31st Mach, 2019, and for school C between 15th March and 5th April, 2019. This means that the period for data collection was 9 weeks (3

weeks for each of the three schools A, B, and C). To solicit views from the teachers and learners, I collected data as indicated below:

Qualitative data was first collected through lesson observations. I observed and documented a total of 15 lessons; that is three lessons for each of the five classes (i.e. class 8V, 8W, 8X, 8Y and 8Z). This means that for each class I observed and documented three lessons (3 lessons per week for each of the five classes) because word problems were schemed and taught at different times (weeks) in the three schools. In order to observe a full lesson, each time I did lesson observation, I sat in the participant's class during their regular mathematics time.

In each class, I first observed the teacher teaching algebraic word problems to grade eights, then I conducted semi-structured interviews with them, and did a document analysis. After that, I conducted FGDs with the five learners who were sampled from each class with the help of the teachers. I followed this order because I wanted to observe the teachers' natural way of teaching algebraic word problems. Interviewing them before being observed would have influenced their teaching practices in a certain way and that could have affected the results of the study. I used FGDs in order to see if what I observed from lesson observations, interviews and document analysis was consistent with what the learners said regarding teachers' classroom practices. Cohen, Manion and Morrison (2004) advised that a focus group usually comprises five to eight individuals who share certain characteristics which are relevant for the study. The case study approach makes use of multiple methods of data collection such as interviews, document reviews, archival records, and direct and participant observations and subsequently 'think descriptions' of the phenomena under study (Yin, 2003). For lesson observations, I captured videos and snapshots while for semi-structured interviews and FGDs I did some audio recording. This helped me during data analysis as I could revisit data each time I needed clarity. Having looked at the data collection and procedure for this study, in the next section I now discuss how data was analysed.

3.10 Data analysis

Data analysis started during the collection exercise by arranging the data collected in line with research objectives. Dawson (2002) stated that, for qualitative data, the researcher might analyse as the research progresses, continually refining and

reorganising in light of the emerging results. Data collected through lesson observations, semi-structured interviews, document analysis and FGDs were analysed vertically according to each instrument and horizontally across all the research instruments in line with the research questions, and themes were generated considering horizontal analysis (Creswell, 2014). Data collected in this study was analysed thematically (McMillan and Schumacher, 1993). During analysis, data recorded from lesson observations, interviews and FGDs was transcribed, edited, coded, categorised, and tabulated according to the research questions (Kothari and Garg, 2014). Data which was answering research question one on how teachers engaged grade eight learners in reading algebraic word problems were manually coded with a numerical value of 1. Data answering research questions two and three on how teachers probed grade eight learners' understanding and how they enhanced learners' process skills and encoding abilities were coded as 2 and 3 respectively.

Data from video lessons, audio recordings of semi-structured interviews and FGDs, as well as from document analysis were interpreted using Newman's (1977) performance strategies. This was done by looking at how teachers probed learners' reading abilities, comprehension abilities, transformation abilities, process skills and encoding abilities in algebraic word problems. Triangulation of data was achieved by comparing what the teachers did when teaching in class, what they said during interviews, what I observed when conducting document analysis, and what the learners said in FGDs. This horizontal analysis of data helped me to generate themes according to the research questions. Data was coded according to research questions as already alluded to and then categorised according to similar ideas patterning to research questions. Themes were then generated based on ideas categorised. Table 2 shows an example of how themes were generated from coded and categorised data during data analysis.

Table 2: *Example of how themes were generated during data analysis*

Lesson Observations	**Semi-structured interviews**	**FGDs**	**Document analysis**
• **T2B** giving chance to grade eight learners to read the questions. (1) • **T1C** read the question for grade eight learners. (1) • **T2A** read the question for grade eight learners. (1) • **T1A** reading the examples for grade eight learners. (1) • **T1B** explaining the meaning of algebraic word problems to grade eight learners. (2) • **T2B** giving chance to learners to explain the meaning of algebraic word problems in their own words. (2)	• **T2A** it is important for learners to read and understand the questions. (1) • **T1A** learners need to read and interpret the question before answering. (1) • **T2B** learners need to understand the questions before solving. (2) • **T1C** learners need to understand a given problem or sentence and also considering the variables. (2)	• **L2C** teachers reading the questions for us. (1) • **L1B** we need to read the question and understand it, and follow algebraic rules. (1) • **L7B** reading and understanding the questions by thinking creatively and then give answers. (1) • **L6A** we don't understand the questions and like and unlike terms. (2) • **L3C** failing to interpret difficult words from the questions. (2)	• **T1A** learners to listen attentively to teachers' explanation. (1) • Not explicitly indicated on how teachers intended to engage learners in reading algebraic word problems. (1) • Not explicitly highlighted on how teachers were going to probe learners' understanding of algebraic word problems. (2) • Though not explicitly done, lesson plans indicated some correct way of presenting answers to algebraic word problems. (3)

After coding and categorising data as shown in Table 2, two themes for the first research question on how teachers engaged grade eight learners in reading algebraic word problems were generated as; giving learners chance to read the questions, and reading the questions for the learners. The following section addresses credibility and trustworthiness of the study.

3.11 Credibility and trustworthiness of the study

In order to ensure credibility and trustworthiness, participants' own words and snapshots of teachers, learners and documents were used in the presentation of findings (Shenton, 2004). Moreover, the themes that emerged after data analysis were subjected to expert view to see whether they were in line with recordings, and recognisable (Merriam, 1998; Adler, 1996). Thus my supervisor and colleagues cross examined them to ensure that they aligned with the study, and if they did not make sense alternative themes were suggested. According to Shenton (2004) credibility and trustworthiness can be attained by adopting research methods which are well established in qualitative approach such as observation, focus groups and individual interviews. In this view, I used these methods in data collection. Moreover, the use of lesson observations, semi-structured interviews, FGDs and document analysis helped me to triangulate data thereby reducing the effect of investigator bias (Maxwell, 1992; Shenton, 2004). I also gave participants opportunities to refuse to participate in the study so as to ensure that data collection sessions involved only those who were genuinely willing to take part and prepared to offer data freely (Shenton, 2004). The next section addresses the ethical issues of this study.

3.12 Ethical considerations

Ethical considerations are an important aspect of any study in that they establish trust between the participants and the researcher. According to Henning (2004, p.73), "human beings are the objects of study in social sciences who need to know their privacy and have humanity protected. Thus, they have the right to know what is going to happen to the information they provide during data gathering". In this regard, I obtained clearance from the Ethics Committee of the University of Zambia and the Department of Mathematics and Science Education (see Appendix A for details, p.118). I also observed originality, quality and ownership of data as well as honesty

(Creswell, 2014). Having been allowed by the University of Zambia to go out for research, I sought the authority of the District Education Board Secretary (DEBS) for Isoka district, to gain entry into schools (see Appendix B for more details, p.119). Informed consent of respondents was sought (see Appendix G for more details, p.128) before proceeding with the research. All the participants were made aware of the nature and purpose of the study, as well as the benefits of participating in the study such as improving the teaching and learning of algebraic word problems and improving the results in algebra, other topics where the knowledge of algebra is required and in mathematics as a whole.

To guarantee the anonymity of participants, I disassociated names from responses during data coding and used pseudonyms where teachers from school A were coded as T_1A and T_2A, and those from school B as T_1B and T_2B, and the one from school C as T_1C. Learners were coded as L_1A to $L_{10}A$ from school A, L_1B to $L_{10}B$ from school B, and L_1C to L_5C from school C, so as to protect their identities (Creswell, 2014). The faces of the participants were not captured in the final document. Moreover, I never shared the information with anyone else apart from individual participants, my supervisor, and scholarly dissemination forums. In addition, the teachers were informed that their participation would not affect their teaching performance in any way, and learners were also assured that it would not affect them as the data collected was purely meant for research purposes. I now give the summary of the chapter.

3.13 Summary of the chapter

In a nutshell, this chapter outlined and described the methodology used in the study. The study used a constructivist research paradigm, and a descriptive case study design following the qualitative approach. Purposive sampling, homogenous in nature was used to select a total of 30 participants (i.e. five teachers and twenty five learners) who participated in the study. Data collection was done by observing teachers first, followed by conducting interviews with them, then by conducting document analysis, and lastly conducting FGDs with learners. Data analysis was done thematically using Newman's (1977) performance strategies. Credibility and trustworthiness was ensured by my supervisor and colleagues. They cross-examined the data and ensured that themes generated aligned with the study. Trustworthiness was also ensured by using

participants own words; writings; snapshots of what transpired during teaching; and excerpts from documents in the presentation of findings as well as by triangulation. To adhere to ethical considerations, permission was sought from the various authorities and from the participants. The next chapter serves to present the study findings according to the research questions.

CHAPTER 4

FINDINGS

4.1 Overview

The previous chapter described the research methodology used in the study to come up with the results which are presented in this chapter. The themes that are presented in this chapter emerged from the data collected from lesson observations, semi-structured interviews, document analysis and focus group discussions (FGDs). In this particular study, data analysis for all the three research questions was aided by Newman's (1977; 1983) performance strategies or error analysis model. Data analysis procedure in this study was specifically guided by the following research questions:

(i) How do teachers engage learners in reading algebraic word problems?
(ii) How do teachers probe learners' understanding of algebraic word problems?
(iii) What do teachers do to enhance learners' process skills and encoding abilities in algebraic word problems?

The themes that emerged from lesson observations, interviews, document analysis as well as FGDs were put under any of the three research questions depending on suitability. Findings for research question one revealed that teachers engaged grade eight learners in reading algebraic word problems in two ways; either learners were given chance to read or the teacher read for them. For research question two, findings showed that teachers probed learners' understanding of algebraic word problems in four ways: giving grade eight learners chance to explain the meaning of the questions in their own words; explaining the meaning of questions to learners; putting learners in small groups to discuss questions and report their solutions; and allowing learners to go direct into solving without probing their understanding. For research question three, findings revealed that teachers enhanced grade eight learners' process skills and encoding abilities by; writing down key information from algebraic word problems when solving, though some teachers did not do that. Some teachers allowed learners to attempt questions on the board. Teachers went round marking and asking learners with problems to explain their reasoning of the processes they used. Some teachers presented final answers to algebraic word problems correctly while others did it

partially correctly. Some teachers were not strict with how learners expressed their final answers, hence they marked correct a variety of solutions to the same questions in many cases, some of which were wrongly expressed. These findings are evidenced by snapshots, excerpts and written work.

The following codes have been used in this study for identification of the participants; T_1A and T_2A (Teacher 1 and teacher 2 at school A respectively), T_1B and T_2B (as teacher 1 and teacher 2 at school B respectively), T_1C (teacher 1 at school C) and learners $L_1A, L_2A, L_3A, \ldots\ldots L_{10}A$ (learner 1, 2, 3,…,10, at school A), $L_1B, L_2B, L_3B, \ldots\ldots L_{10}B$ (learner 1, 2, 3, ……,10, at school B) and $L_1C, L_2C, L_3C \ldots\ldots L_5C$ (as learner 1, 2, 3,…..,5, at school C).

4.2 Research question 1: How do teachers engage learners in reading algebraic word problems?

Research question one focused on observing how teachers engaged grade eight learners in reading algebraic word problems. To ensure that learners have abilities in reading, teachers need to see to it that learners can recognise words and symbols used in algebraic word problems. Firstly, teachers were observed teaching algebraic word problems to grade eight learners, thereafter interviews were conducted with them and document analysis was done in order to see if they planned on how to engage learners in reading algebraic word problems. Hence, in this section, the analysis of teachers' classroom practices was gauged on how they engaged grade eight learners in reading algebraic word problems and if at all they paid attention to word and symbol recognition by the learners as they read the questions. The findings revealed that teachers engaged learners in reading algebraic word problems in two ways; either grade eight learners were given chance to read or the teacher read for them.

4.2.1 Giving chance to learners to read algebraic word problems

Results from lesson observations revealed that one (1) out of five (5) teachers gave chance to individual grade eight learners to read algebraic word problems first. This teacher called upon a good number of learners to read and reread the algebraic word problems aloud to the class before solving. The teacher who did this only read through the questions aloud to the class after a number of learners had read through. This

enabled the teacher to identify learners with reading problems and those who had no problems at all. Learners with reading problems were assisted regarding the correct pronunciation of the words and symbols in the problems. The following snapshots (Figure 4) highlight examples where the teacher (T_2B) first gave chance to grade eight learners to read algebraic word problems aloud to the class:

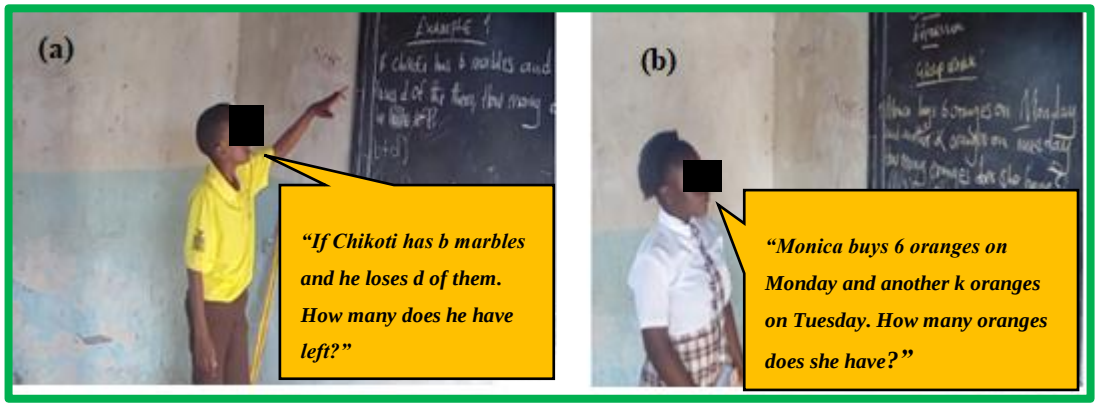

Figure 4: *Grade eight learners reading algebraic word problems aloud to the class in* $\boldsymbol{T_2B's}$ *class in both panels a and b.*

4.2.2 Reading the questions for learners

Research results from lesson observations showed that four (4) out of five (5) teachers first read the questions for the class rather than engaging grade eight learners themselves in reading. After reading algebraic word problems, they only engaged learners with chorus reading. However, interviews with teachers revealed that they knew how important it was for grade eight learners to have reading abilities as it was crucial in solving algebraic word problems. This came out when I asked them the kind of knowledge learners required to have for them to solve algebraic word problems easily, and what they thought was a major drawback in as far as learners' solving of algebraic word problems was concerned. For instance, the following extracts from teachers (T_1A, T_1B and T_1C) are the typical examples of their responses as shown below:

> *"Word problems are difficult for the learners who do not know how to read"* ($\boldsymbol{T_1A}$). *"Learners need to have knowledge of how to read, understand and interpret algebraic word problems for them to solve easily"* ($\boldsymbol{T_1B}$). *"Learners should be able to read and understand a given problem or sentence and also consider the variables used"* ($\boldsymbol{T_1C}$).

The following snapshots (Figures 5) highlight examples where teachers first read algebraic word problems for the learners:

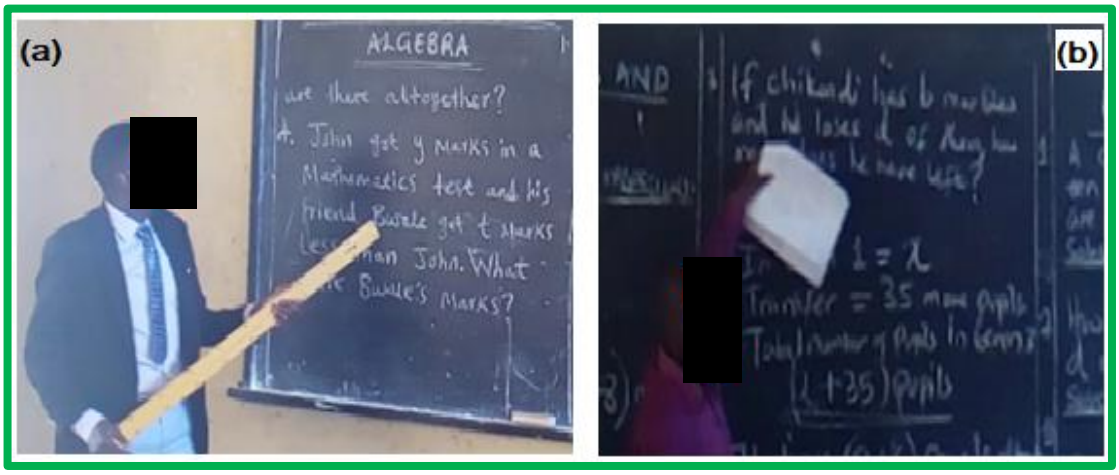

Figure 5: *Reading examples for the learners by* $\boldsymbol{T_2A}$ *(panel a) and* $\boldsymbol{T_1A}$ *(panel b).*

The following are the excerpts of teachers (T_2A and T_1A) reading algebraic word problems for grade eight learners as shown in Figure 5 above:

> *"John got y marks in a mathematics test and his friend Bwale got t marks less than John. What were Bwale's marks?"* $\boldsymbol{(T_2A)}$.
>
> *"If Chikondi has b marbles and he loses d of them, how many does he have left?"* $\boldsymbol{(T_1A)}$**.**

Findings from document analysis which was done by looking at the lesson plans for teachers in the study revealed that the aspect of giving grade eight learners chance to read algebraic word problems in class was not explicitly planned for by all the teachers. Teachers did not explicitly show in their planning how they were going to engage learners in reading so as to discover the errors that learners make regarding word and symbol recognition, despite having the lesson plans in place. However, word and symbol recognition though not explicitly highlighted, were implied in the lesson plans by two (2) out of five (5) teachers by the way they wrote examples. But still this was not enough as it did not involve learners bringing out what they knew regarding reading of algebraic word problems. The following snapshots (Figure 6) highlight examples of this fact:

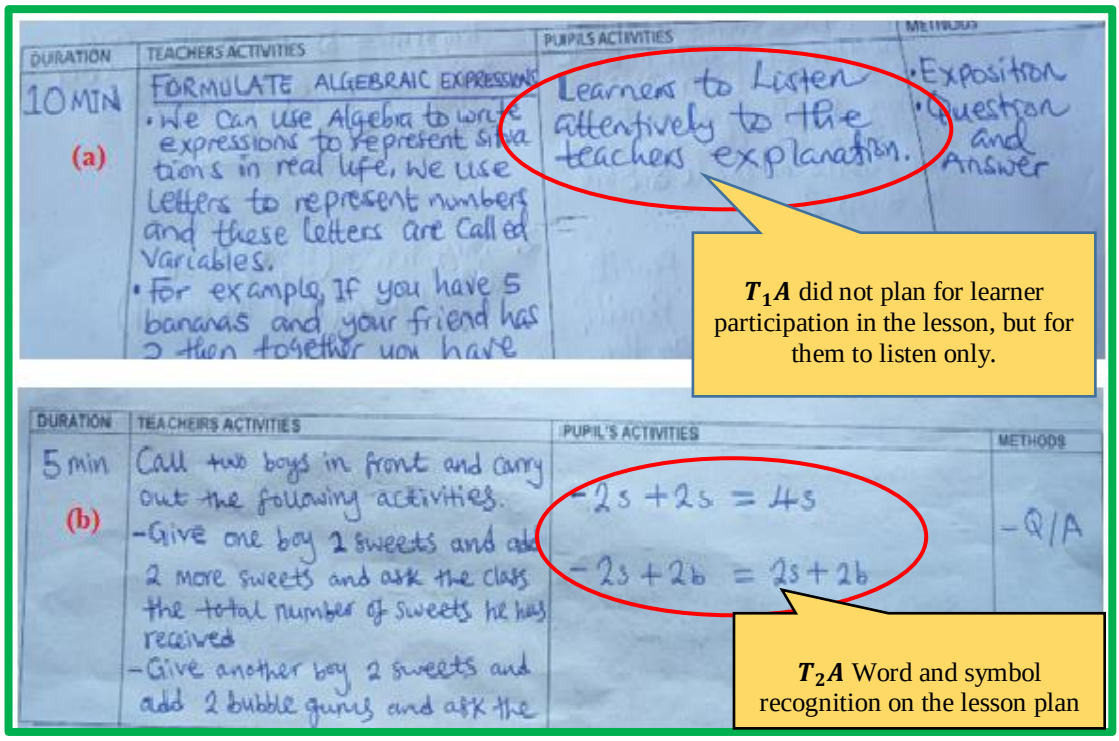

Figure 6: *Sample lesson plans for* $\boldsymbol{T_1A}$ *and* $\boldsymbol{T_2A}$ *in panels a and b respectively.*

During FDGs, learners also acknowledged the fact that reading and understanding was crucial for them to solve algebraic word problems easily. For instance when I asked them what knowledge they needed to have in order for them to solve algebraic word problems easily; the following extracts by learners (L_2A and L_6B) are typical examples:

> *"We need the knowledge of reading and understanding the questions by thinking creatively and then give answers"* ($\boldsymbol{L_6B}$). *"We need the knowledge of understanding the question, identifying and understanding the words in the sentence"* ($\boldsymbol{L_2A}$).

4.3. Research question 2: How do teachers probe learners' understanding of algebraic word problems?

Research question two focused on observing how teachers probed grade eight learners' understanding of algebraic word problems. Understanding algebraic word problems in this case involved looking at what teachers did to probe learners' comprehension and transformation abilities in algebraic word problems. In order to do that, I observed teachers teaching algebraic word problems to grade eight learners, thereafter I conducted interviews with them and document analysis was done in order to see if

probing learners' understanding of algebraic word problems was planned for by the teachers.

4.3.1 Comprehension abilities (Literal and specific terminology understanding)

In this section, the analysis of teachers' classroom practices was gauged on how they probed grade eight learners' understanding of algebraic word problems and if at all they paid attention to literal and specific terminology understanding of algebraic word problems by learners. The following themes emerged:

4.3.1.1 Giving learners chance to explain the meaning of the questions

Research results from lesson observations showed that one (1) out of five (5) teachers asked learners to explain in their own words, the meaning of algebraic word problems after reading. The same teacher who gave learners chance to read algebraic word problems also gave individual learners chance to explain in their own words the meaning of algebraic word problems at hand. In their various explanations, some learners could not do it adequately. By so doing, the teacher noted some errors and asked capable learners to explain for those who could not explain properly. Hence, learners' errors in terminologies understanding were noted and corrected. The snapshots below (Figure 7) highlight examples where the teacher (T_2B) gave chance to grade eight learners to explain in their own words the meaning of algebraic word problems to the class.

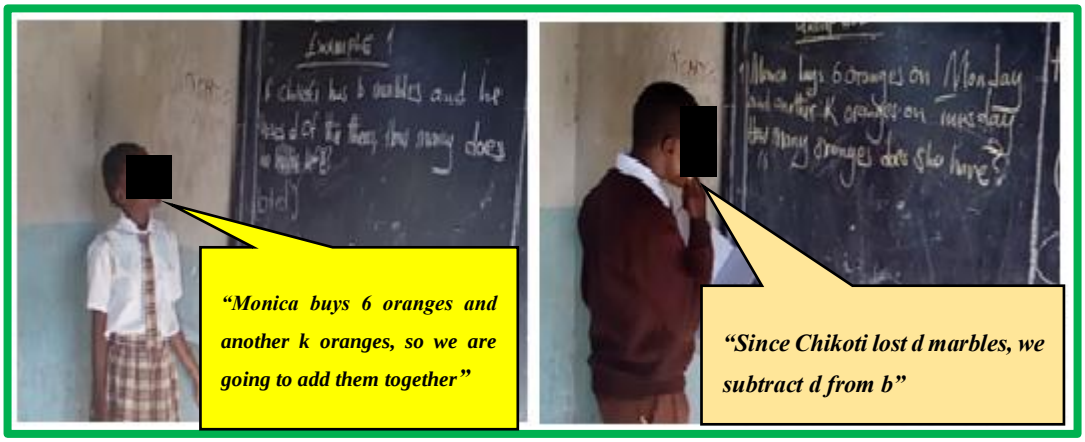

***Figure 7**: Learners explaining the meaning of algebraic word problems in their own words in **T_2B's** class.*

This teacher again noted learners' errors in their explanations and corrections were made when discussing the questions with them after they attempted to explain in their own way. The following extract in form of a conversation in T_2B's class is a typical example where learners' errors were noted in their explanations and corrected through further questioning:

> ***T_2B***: *If Chikoti has b marbles and he loses d of them, how many does he have left? Who can come and explain this one?*
>
> ***Learner 1****: We should add b and d to find the number of marbles.*
>
> ***T_2B***: *Thank you for attempting. Anyone with a different explanation?*
>
> ***Learner 2:*** *We should subtract b minus d.*
>
> ***T_2B***: *Can you explain why we should subtract?* (Learner 2 was silent)
>
> ***T_2B***: *Who can come and explain?*
>
> ***Learner 3:*** *We subtract because he had b marbles and he loses d marbles.*
>
> ***T_2B***: *Thank you! Who can shade more light?*
>
> ***Learner 4:*** *I think b minus d is correct because on my own I know that in mathematics if we use loses we have to subtract.*
>
> ***T_2B***: *Thank you very much! Clap for them!* (Learners clapped).
>
> ***T_2B***: *Have you all understood now?*
>
> ***Learners:*** *Yes sir!*
>
> ***T_2B***: *Ok, that's good, let us now move to another example.*

4.3.1.2 Explaining the meaning of the questions to the learners

Results from lesson observations showed that four (4) out of five (5) teachers explained the meaning of algebraic word problems for the grade eight learners without first asking them to explain the meaning of the questions in their own words. Teachers who first read questions for the learners also explained the meaning of the algebraic word problems for them without finding out if learners could do it on their own. The following snapshots highlight examples where teachers explained the meaning of algebraic word problems to grade eight learners (Figure 8 (a) and (b)).

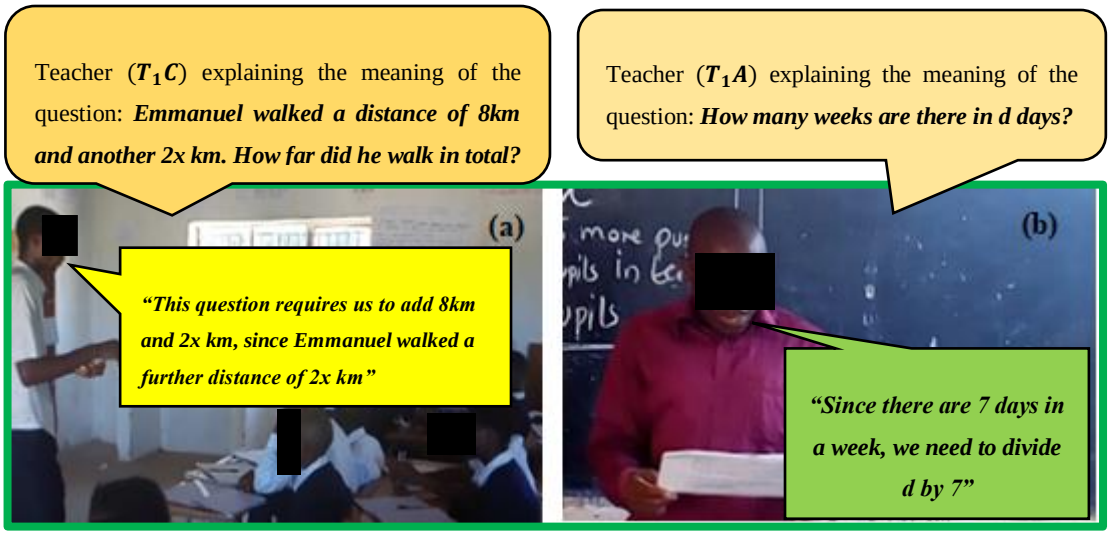

Figure 8: *Teachers explaining the meaning of questions to grade eight learners* T_1C *(panel a) and* T_1A *(panel b).*

Results from document analysis showed that there was no planning of how teachers intended to probe learners' understanding of algebraic word problems when teaching. The lesson plans of all the teachers did not highlight deliberate strategies aimed at giving learners chance to explain the meaning of algebraic word problems in their own words before answering. Some teachers indicated in lesson plans that learners were only going to listen carefully to the teachers' explanation (see Figure 6(a) for evidence, p.55).

4.3.1.3 Putting learners in small groups to discuss the questions

Teachers (2 out of 5) put learners in small groups where they discussed algebraic word problems and some groups were made to report their answers to the class. During small group discussions, learners discussed the meaning of algebraic word problems in their respective groups and explained when presenting their answers to the whole class. The following snapshots highlight examples where teacher (T_1B) put grade eight learners into small groups to discuss algebraic word problems (Figures 9 (a) and (b)):

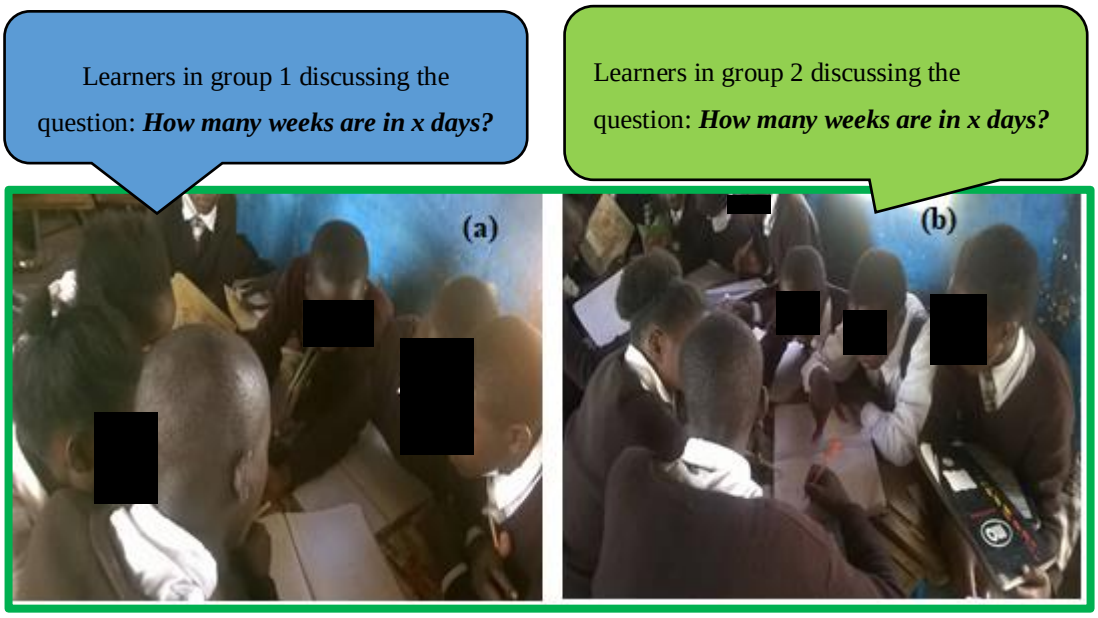

Figure 9*: Grade eight learners discussing algebraic word problems in groups in* $\boldsymbol{T_1B's}$ *class, Group 1(panel a) and Group 2 (panel b).*

To explicate on the group discussions, I present the following extracts of learners in group 1 as they discussed the question, '*How many weeks are there in x days?'*

> ***Learner 1:*** *"*$x = 7$*, since there are 7 days in the week."*
>
> ***Learner 2:*** *"No,* x *is not equal to 7 because we have to add* $x + 7$ *to find the number of weeks in* x *days."*
>
> ***Learner 3:*** *"I think to find the number of weeks in x days, we divide 7 by x."*
>
> ***Learner 4:*** *"But the number of days is x, so I think we should divide x by 7 to find the number of weeks."*
>
> ***Learner 3:*** *"Why dividing x by 7 and not 7 by x?"*
>
> ***Learner 4:*** *"Because x days should be put in groups of 7 days to find the number of weeks, since there are 7 days in a week."*
>
> ***Learners 3:*** *"oh I see! You are right!"*

The conversation for group 2 was similar to that of group 1, hence, that of group one is given as a typical example of what transpired in the groups.

Some groups were then randomly sampled by the teachers to present their solutions to the whole class through the group representatives. The following snapshots highlight

examples where teacher (T_2B) gave chance to group representatives to report to the class their discussions and answers to the questions (Figures 10 (a) and (b)):

Figure 10: *Group representatives sharing their respective discussions with the entire class in* $\boldsymbol{T_2B's}$ *class, Group 2 representative (panel a) and Group 4 representative (panel b).*

In figure 10 (a), the learner (L_6B)was explaining the question for group 2 which was, *'Monica buys 6 oranges on Monday and another K oranges on Tuesday. How many oranges does she have?'* The following extract was his explanation:

> *"We are going to use plus because she bought some more oranges on Tuesday. So the answer will be (6 + K) oranges" (*$\boldsymbol{L_6B}$).

After explaining the solution to the question, the teacher asked the class if they had any questions for group 2. The class had no questions and so the learners were told to clap for the group and the learner (L_6B) was told to go and sit.

The teacher (T_2B) then sampled group 4 to present the same question after group 2 had presented. Figure 10 (b) shows the representative of group 4, learner (L_3B) explaining the question. The following extract was her explanation:

> *"Since Monica buys 6 oranges and another K oranges on Tuesday, so we have to add because she bought some more. So the answer is (6+K) oranges" (*$\boldsymbol{L_3B}$).

At first, the learner (L_3B) made an error and wrote (6 + 4) oranges instead of (6 + *K*) oranges, mistaking 4 for *K*. The instant reaction by the class made her realise she had made an error, she then rubbed the 4 and wrote the correct answer as (6 + *K*) oranges.

The following snapshot (Figure 11) shows an error made by (L_3B) in writing the answer to the above question at first, though her explanation was correct:

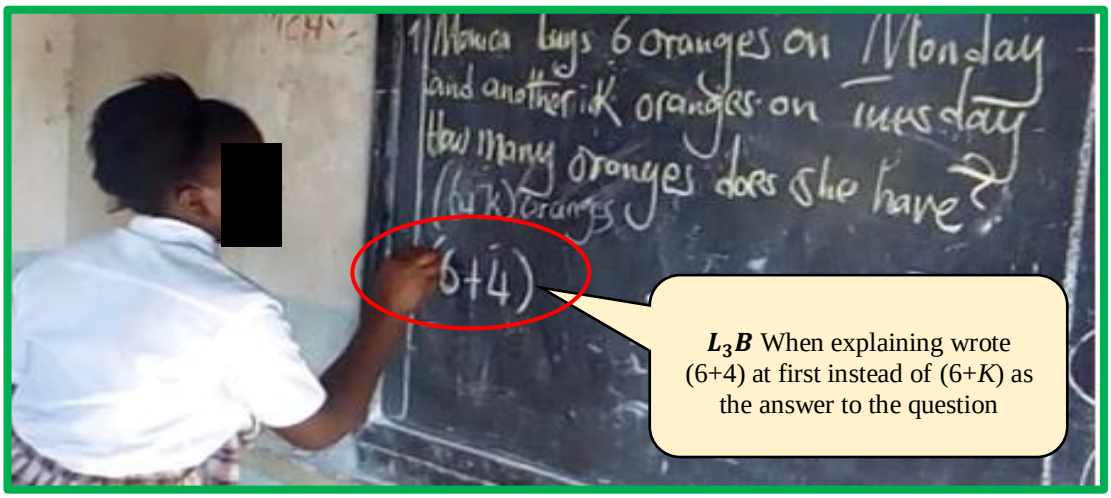

Figure 11: *Learner (L_3B) committing an error in expressing the final answer to the question despite having explained well.*

After group 4 presented, the teacher again asked the class if they had any questions, to which the answer was again no. The teacher then asked the class to clap for the group and told the presenter to go and sit. After that was done, the teacher asked if there was any group with a different answer and the learners said no. It was at this point that he started writing the exercise on the board and told the learners to start copying the exercise in their exercise books.

4.3.1.4 Allowing learners to go direct into solving without showing some understanding

Results from lesson observations also showed that one (1) out of five (5) teachers allowed grade eight learners to go straight into answering algebraic word problems without showing some understanding by verbal explanations. The following was the verbatim by the teacher (T_1C) after writing the first two examples on the board:

> *"Someone to come and answer example one on the board….anyone willing to answer the first question? I want to see how you can answer it."* ***(T_1C)***.

Findings from semi-structured interviews showed that teachers actually knew that some grade eight learners failed to solve algebraic word problems, apart from failing to read, because they did not understand the questions and the specific key terms used.

They answered this when I asked them what they thought was the most significant problem solving difficult for learners to solve algebraic word problems. The following were the extracts by teachers (T_1A, T_2A andT_2B) as representative examples:

> *"Failing to understand questions is a major problem for grade eight learners" (**T_2B**). "Failure to interpret algebraic word problems is a problem for grade eight learners" (**T_1A**). "Algebraic word problems are difficult for grade eight learners because they fail to interpret words to arithmetic" (**T_2A**)*

In FDGs grade eight learners stated that some of them had problems in answering algebraic word problems mainly because of not understanding the problems written in words. They said this when I asked them of the difficulties they were facing when answering algebraic word problems. The following were the excerpts by learners (L_2A, L_5B and L_7B) as typical examples:

> *"I have the problem of understanding what the question says and knowing the meanings of words in the question" (**L_2A**). "We face difficulties in interpreting difficult words" (**L_5B**). "I have difficulties in signs interpretation" (**L_7B**).*

4.3.2 Transformation abilities

For teachers to probe learners' understanding of algebraic word problems, they also needed to probe learners' abilities to select mathematical processes which were required to find an answer to the given problem.

4.3.2.1 Asking learners to translate algebraic word problems

It was also observed that the same teachers (1 out of 5) who gave grade eight learners chance to read and explain the meaning of the questions in their own words also gave them chance to state the processes required to be performed in order to get a correct solution. The teacher who did this also allowed a good number of learners to give out their views on translation of questions and also on the processes required to answer the questions. This teacher only came in to discuss with the learners when it was time to decide which of all the attempts by learners was correct. The following conversation is a typical example where the teacher (T_2B) gave learners chance to translate the questions:

'*A school has 600 learners enrolled at the start of the school year. During the year an additional z learners were enrolled. How many learners were enrolled altogether?'*

T_2B: *What do you need to know for you to solve this question?*

Learner 1*: To know that there are 600 learners in school in term one.*

T_2B: *What else do you need to know?*

Learner 1*: ….that additional learners of z were enrolled also.*

T_2B: *What does additional mean? How can you say it in your own words?*

Learner 2: *let me try….additional means to add.*

T_2B: *Ok, what do others say?*

Learner 3: *It means putting some more to what is already there.*

T_2B: *Good! So in this case what does the question requires you to do?*

Learner 3: *To add together 600 and z.*

T_2B: *Very good! Is that clear everyone?*

Learners: *Yes sir!*

Results from semi- structured interviews indicated that teachers were actually aware of the fact that learners needed to be given time to explore algebraic word problems in order for them to develop conceptual understanding. This came out when I asked them on what strategies were favourable for the learners to understand algebraic word problems. The following excerpts by teachers (T_2A andT_2B) are typical examples:

> *"Learning in abstract make it difficult for the learners to understand the concepts, therefore I involve them in learning" (**T_2A**). "I make teaching more learner centred by letting learners do it on their own" (**T_2B**).*

4.3.2.2 Translating algebraic word problems for the learners

Results from lesson observations showed that the same teachers (4 out of 5) who read and explained algebraic word problems for the learners, also did not ask them to translate the algebraic word problems and to outline the mathematical processes which were needed to find the answers, but instead they did that for the learners. These teachers only engaged learners with chorus answers at some point in their explanations. The following excerpts by teachers (T_1B and T_1C) are typical examples of how teachers translated algebraic word problems for the grade eight learners:

Question: Njabi is 15 years old. Write down an expression for her age: (a) *x* years ago (b) in *x* years time.

> *"This question in other words is asking what Njabi's age was some x years ago in part a, and what his age will be in x years to come in part b. So this means we subtract in part a and add in part b"* (***T_1B***).

Question: Sitali walks a distance of *3km* and then another *q* km to get home from school. How far is this?

> *"This question means the same as Sitali moves a distance of 3km and another q km to reach home when coming from school. So it means that there is need for us to add the two distances which Sitali walks when going home from school"* (***T_1C***).

4.4 Research question 3: What do teachers do to enhance learners' process skills and encoding abilities in algebraic word problems?

Research question number three focused on observing what teachers did to enhance the process skills and encoding abilities of grade eight learners in algebraic word problems. For process skills, teachers needed to ask learners to verbalise the processes required to gain an answer, or to see the learners performing the actual processes of solving algebraic word problems as outlined at transformation level. For encoding abilities, teachers needed to look at how learners expressed the final answers and their competences in words (spellings) and how they used symbols. Teachers needed to ensure that grade eight learners followed the processes and wrote the final answers accurately and consistently. To gather data in this regard, teachers were observed teaching algebraic word problems to grade eight learners, thereafter semi-structured interviews were conducted with them and document analysis was done in order to see if that was also planned for.

4.4.1 Process skills

4.4.1.1 Writing down key information from word problems

The results of lesson observations revealed that one (1) teacher out of the five (5) teachers wrote down the key information on the board before solving algebraic problems. The teacher extracted key information from algebraic word problems which guided the process of expressing the final answer. The following snapshot below

(Figure 12) is a typical example where the teacher (T_1A) highlighted the key information which could help in solving algebraic word problems from the given question when explaining to grade eight learners:

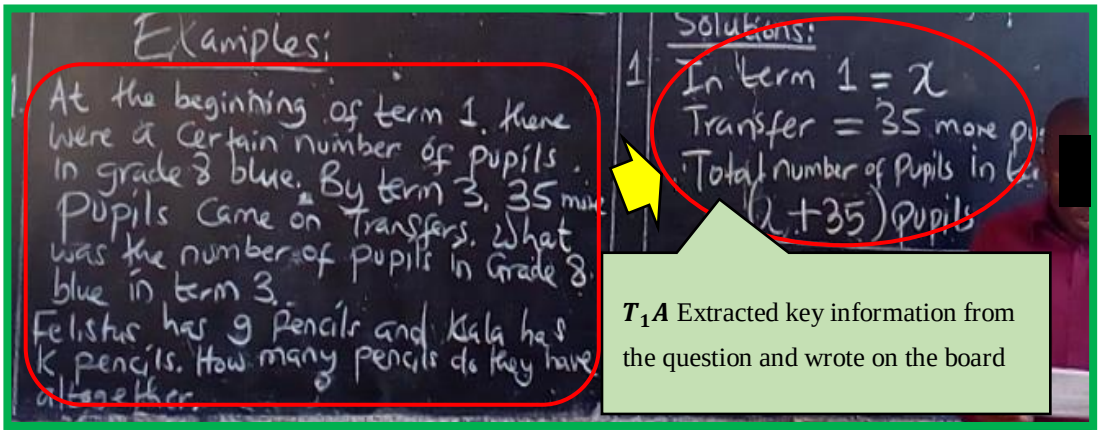

Figure 12: *Processes of solving algebraic word problems written on the board by* $\boldsymbol{T_1A}$.

The following excerpts shows how the key information were extracted from the word problem shown on the board in figure 12 by the teacher ($\boldsymbol{T_1A}$):

> *"Let the number of pupils in term one be x, since it is unknown…in term three 35 pupils came on transfer. This is the information you need to solve this problem….total number of pupils in term three is equal to (x + 35) pupils"* ($\boldsymbol{T_1A}$).

4.4.1.2 Not writing down key information from word problems

Lesson observations also showed that four (4) out of five (5) teachers did not write down the key information from algebraic word problems when solving on the board. These teachers only showed the final answers without showing or encouraging learners to extract some key information from the given problems before writing the final answers.

The following excerpts provide how teachers (T_1B and T_1C) in figures 13 (a) and (b) answered the questions indicated without writing down key information first respectively:

> *"A chicken has two legs, so we can say the number of chicken legs is 2x, a goat has four legs, so we can say the number of legs is 4y, therefore, the total number of legs for these animals is 2x+4y."* ($\boldsymbol{T_1B}$).

"Chikoti has b marbles and loses d of them, so the answer will be b – d" ***(T_1C)***.

Figures 13 (a) and (b) below are the typical examples of that:

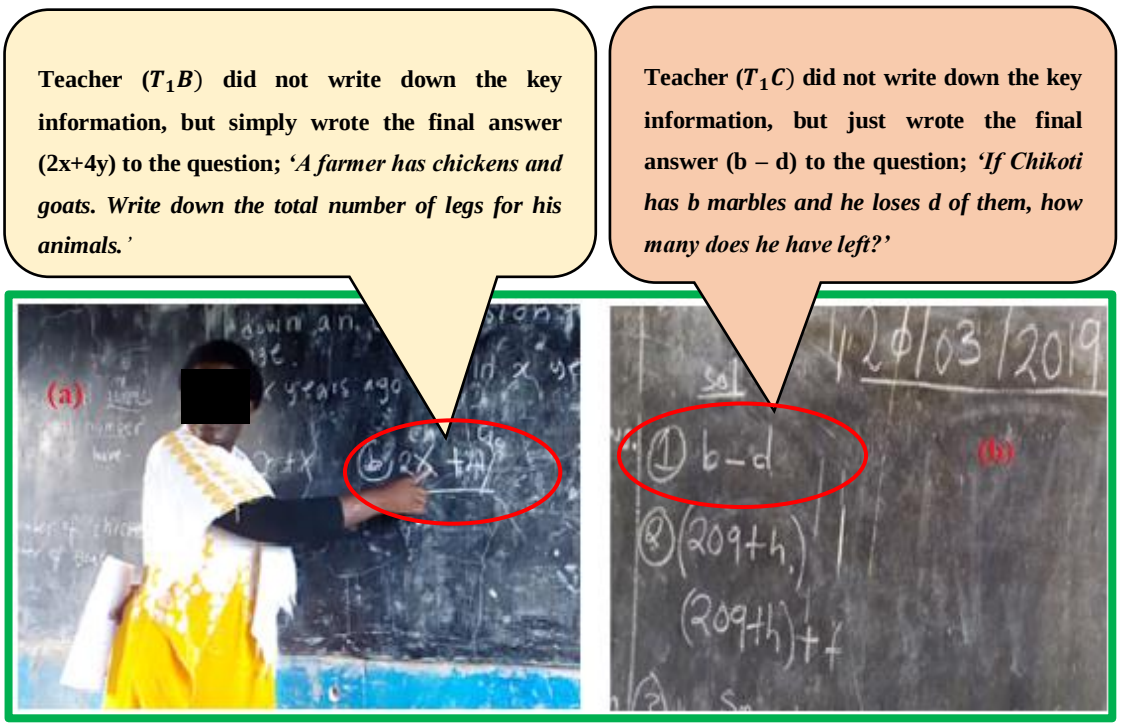

Figure 13: *Processes of solving algebraic word problems by* ***T_1B*** *(panel a) and by* ***T_1C*** *(panel b).*

4.4.1.3 Allowing learners to write answers on the board

The results of lesson observations indicated that teachers (3 out of 5) allowed a good number of grade eight learners to write their answers on the board whether correct or wrong and then discussed those answers with the entire class and concluded on which of these answers was correct. By doing that errors that grade eight learners made through random responses, wrong operations, faulty algorithms, spatial and logic were exposed and teachers worked on these errors and corrected the learners. In this manner, even learners who had no responses at all were also helped as the discussion of all the answers was done. For example the question such as; *'A class has x learners, eighteen of them are boys. How many are girls?'* attracted a number of answers in $T_2B's$ class and about three solutions to the same problem were presented on the board by the learners. Below are the snapshots illustrating this fact: Figure 14 (a) shows the different answers to the same question by grade eight learners and Figure 14 (b) shows the

teacher (T_2B) facilitating the discussions towards coming up with the correct answer amongst the three attempts by the learners.

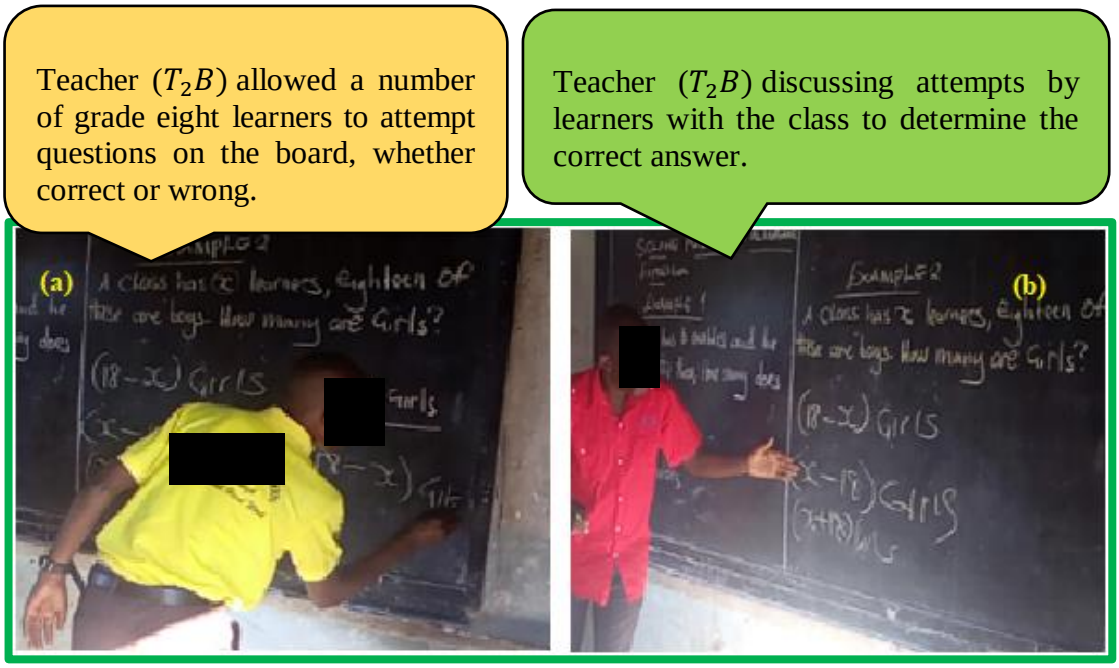

Figure 14: *Different attempts by learners (panel a) and* ***T_2B*** *facilitating the discussions of learners' attempts to determine the correct answer (panel b).*

The following are the excerpts of the conversation which was there before and during the discussion of the attempts which learners wrote on the board:

> ***T_2B***: *A class has x learners, eighteen of these are boys. How many are girls? Who can come and answer this one?*
>
> ***Learner 1:*** Wrote *(18 - x) girls* on the board.
>
> ***T_2B***: *Ok, that is his idea (he called the learner's name). Another one to come and try? I want two or three attempts then we debate the answers.*
>
> ***Learner 2:*** Wrote *(x – 18) girls* on the board.
>
> ***T_2B***: *Ok, now the last one. I want three of them and then we debate.*
>
> ***Learner 3:*** Wrote *(x + 18) girls* on the board.
>
> ***T_2B***: *Ok, now let us discuss the answers. A class has x learners, eighteen of these are boys. How many are girls? Who can come and explain for the first one? Why (18 – x) girls?*
>
> ***Learner 4:*** *I want to explain number 2.*
>
> ***T_2B***: *Even number 2 is fine, explain.*

Learner 4: *They don't tell us the number x but they tell us that eighteen of these are boys, find the girls…that's why he put (x – 18) girls.*

T_2B: *Ok, another one to come and explain?*

Learner 5: *we cannot start with the number because we don't know the number of pupils in class and so the second one (x – 18) girls is correct.*

T_2B: *Very good! Clap for her! The one who wrote (x +18) come and explain.*

Learner 3: *on my own, since here there is plus they don't tell us the number of girls, but they tell us the number of boys so we have to add x and 18.*

Learner 6: *Even me sir, I agree since they want the number of girls we have to add x and 18.*

T_2B: *Thank you, all answers are correct, but we need the best answer. The total number of learners is x and the number of boys is 18, hence the number of girls is (x – 18) girls. So the second one is the correct answer.*

4.4.1.4 Going round marking and asking learners to explain their processes

The results from lesson observations also showed that teachers (2 out of 5) allowed learners to clarify the processes they used when answering algebraic word problems verbally when going round marking exercise books, especially those who had challenges in answering the questions. Figure 15 (a) and (b) show how the teachers (T_1A) and (T_1B) dealt with learners' process skills when going round the classroom marking books respectively.

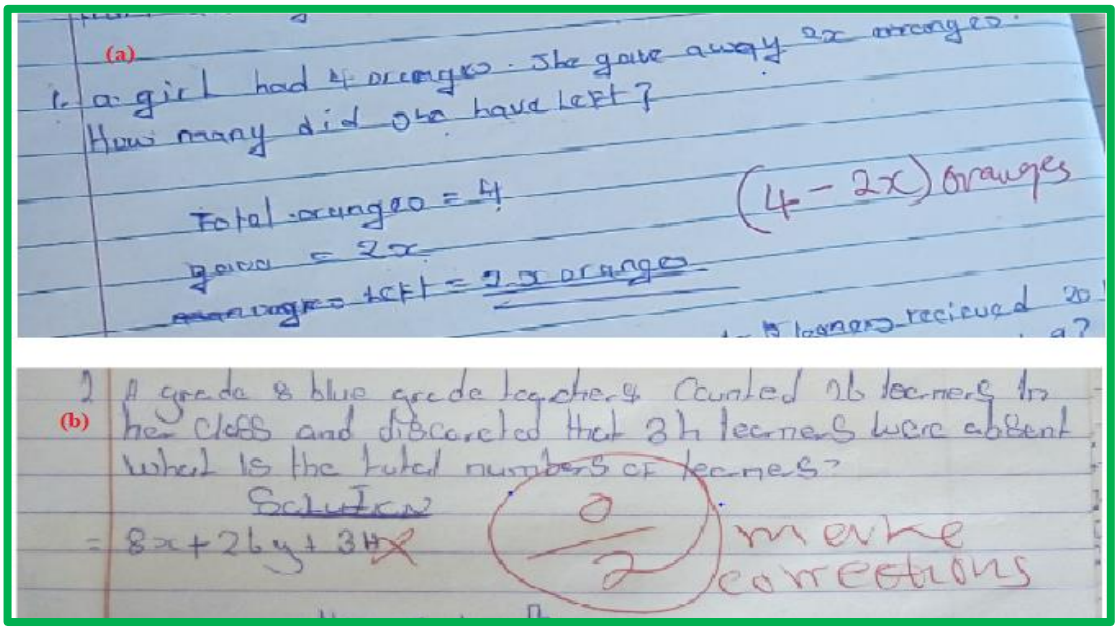

Figure 15: *Dealing with learners' process skills,* ***T_1A*** *(panel a) and* ***T_1B*** *(panel b).*

In Figure 15 (a), the following excerpts represent the conversation between the teacher and the learner on the processes to the question:

'A girl had 4 oranges. She gave away 2x oranges. How many did she have left?'

> $\boldsymbol{T_1A}$: *What have you done to find 2x oranges as the final answer?*
>
> ***Learner:*** *The total is 4 oranges and she gave away 2x oranges and so she remains with 2x oranges.*
>
> $\boldsymbol{T_1A}$: *Why?*
>
> ***Learner:*** *Because 4 minus 2x is 2x oranges.*
>
> $\boldsymbol{T_1A}$: *No you cannot subtract like that because 4 and 2x are not like terms, so the answer will just be (4 – 2x) oranges* (Teacher wrote in the learner's book as shown in the Figure 15 (a) above).

In Figure 15 (b), the following excerpts represent the discussion between the teacher and the learner on the processes to the question, *'A grade 8 blue grade teacher counted 26 learners in her class and discovered that 3h learners were absent. What is the total number of learners?'*

> $\boldsymbol{T_1B}$: *Why have you added 8x, 26y and 3h?*
>
> ***Learner:*** *Because in the question there is 8, 26 and 3h, and the question is asking the total.*
>
> $\boldsymbol{T_1B}$: *No, you can't add like that… you should read the question again and understand what it requires you to do. The 8 is just showing the grade of learners, it is only 26 and 3h which are representing the number of learners. So make corrections, I will come and mark.*

In Figure 15 (a) above, the teacher (T_1A) did not probe the learner's process skills further by asking the learner to write the processes needed to gain a solution. Instead the teacher wrote it for the learner. The teacher only solicited for the learner's verbal explanation. On the other hand, in Figure 15 (b) above, the teacher (T_1B) marked the question wrong and solicited the learner's explanation of the processes but did not ask the learner to write down the required processes also.

However, semi-structured interviews with teachers showed that they knew how important it was for them and learners to show the processes of solving algebraic word problems. The following excerpts are typical examples of teachers' (T_1B andT_1C)

responses when I asked them about the challenges learners face when answering algebraic word problems:

> *"Learners have challenges in writing their work systematically"* (***T_1B***).
> *"Learners fail to interpret and solve questions by showing necessary working"* (***T_1C***).

Teachers also corrected grade eight learners who understood the questions but did not have the ability or had limited ability to write the processes and the answers correctly. Figure 16, is a typical example of that. In the figure, the teacher (T_1A) noted that the learner had the concept but did not have the ability to express the processes needed to be performed to gain a solution. Hence, he highlighted those processes by writing in the learner's book. The question involved finding the perimeter of the square with side *t* centimetres (cm) as it can be seen in Figure 16 (panel a) below. The learner knew that to find the perimeter of a square he needed to add all the four sides of length *t cm* each (Figure 16 panel b). But after expressing it that way he did not realise that they were like terms which needed to be added to get *4t cm*. The learner perhaps forgot that the *ts* could be added as they were like terms, or it could be that he did not know that at all.

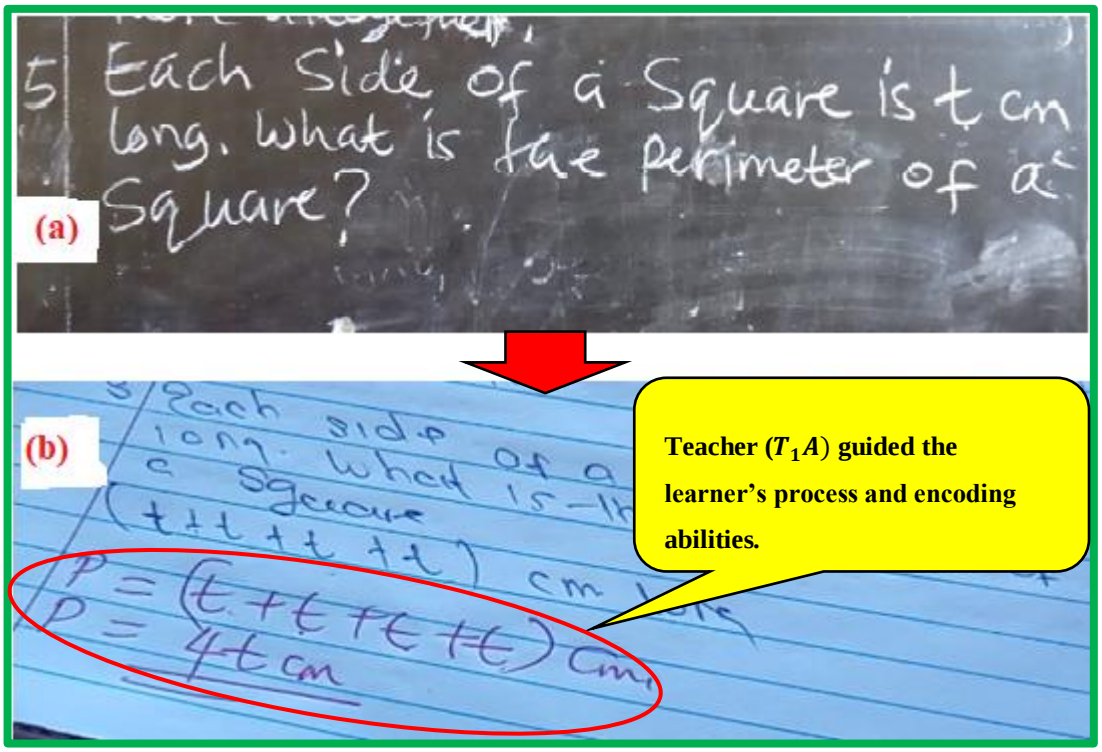

Figure 16: *Dealing with learners' errors in process and encoding by* ***T_1A.***

The following excerpts highlight the dialogue between the teacher (T_1A) and the learner on the processes of the question, *'Each side of a square is tcm long. What is the perimeter of a square?'* as shown in Figure 16 (a) and (b) above:

> ***T_1A***: *How did you know that you were supposed to add the four ts?*
>
> ***Learner:*** *Because the question is asking to find the perimeter of the square of each side t cm…and to find perimeter is to add all the four sides of the square.*
>
> ***T_1A***: *But why have you expressed the final answer as (t + t + t + t) cm long? Don't you think you can add the ts?*
>
> ***Learner:*** *Oh ok…I see! The ts are like terms. I forgot.*
>
> ***T_1A***: *Yes you are right, the ts are like terms and so you should show the process like the way you started, but then show the final answer as 4t cm.*

Results from document analysis revealed that there was no deliberate planning which was done by the teachers on how they were going to deal with grade eight learners' errors in enhancing their process skills when teaching algebraic word problems. The lesson plans of all the teachers did not reflect how they intended to deal with learners' errors concerning the process skills.

Data from FDGs confirmed that teachers did not give enough chance to individual learners to explain the processes required to get the correct answer. This came out from the learners when I asked them about what they thought the teachers needed to do in order for them to understand and solve algebraic word problems easily. The following excerpts by learners (L_9A and L_4B) are typical examples:

> *"The teacher should give us chance to be explaining the questions" (**L_9A**).*
> *"The teacher should give us more home work for us to be solving mathematics every day and also to give us more examples" (**L_4B**).*

4.4.2 Encoding abilities (Word and symbol recognition)

Encoding abilities has to do with how the final answer to the mathematical problem is expressed. Here word and symbol recognition means that learners are expected to express both words and symbols correctly in the final answers to algebraic word problems. I was particularly concerned with what teachers did to enhance learners' encoding abilities and how they paid attention to the learners' final answers to algebraic word problems regarding word and symbol recognition. Hence the difference

on how word and symbol recognition is viewed here from the way it was considered under research question one.

4.4.2.1 Presenting final answers correctly

Results from lesson observations revealed that teachers (3 out of 5) paid attention to the encoding abilities of the learners both on words and symbols very well. These teachers presented the final answers well on the board and therefore, learners followed suit in their writing of the final answers (see Figure 19 for details, p.74). Figure 17 below is a typical example:

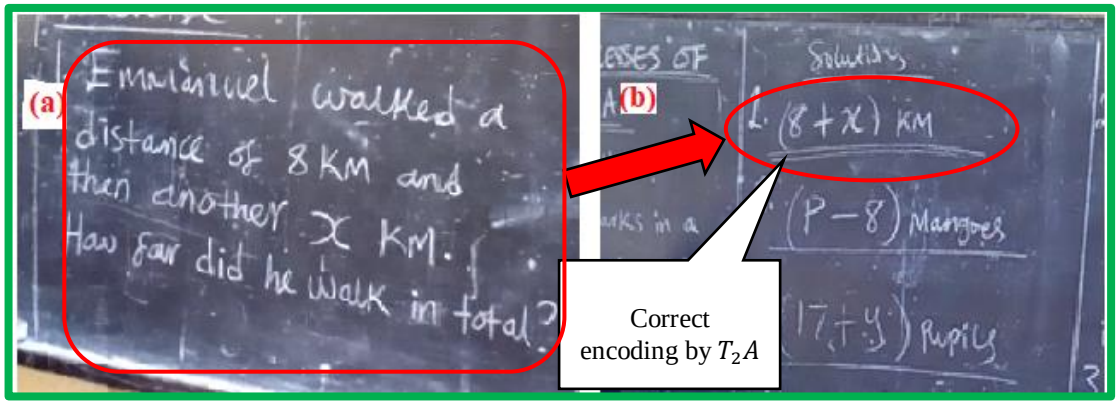

Figure 17: *Correct presentation of the final answers on the board by* $\boldsymbol{T_2A}$.

The following excerpts presents the actual conversation between the teacher and learners during revision of the question in Figure 17 panel (a) which led to the answer on Figure 17 panel (b) as shown above:

> $\boldsymbol{T_2A}$: *Ok, let's go through the question together. Emmanuel walked a distance of 8 km and then another x km. How far did he walk in total? What are we going to do?*
>
> ***Learners:*** *We add…*
>
> $\boldsymbol{T_2A}$: *We add what?*
>
> ***Learners:*** *We add 8 and x.*
>
> $\boldsymbol{T_2A}$: *Yes you are right, we are going to add them and write the answer as (8 + x) km.*

The teacher (T_2A) discussed the rest of the questions with the class and encoded the solutions to them correctly just like the way he did for the question in Figure 17 (a).

For these teachers who expressed their final answers correctly on the board, even on their lesson plans the encoding aspect was properly done though there was no indication in planning as to how they were going to deal with grade eight learners' encoding abilities in order to avoid errors. In their writing of the final answers on their lesson plans, they indicated correctly the digits, variables, symbols and words. Because of their consistence in encoding, very few learners in their classes made encoding errors. These teachers considered the correct encoding when marking the books and corrected learners' errors in encoding there and then. Figures 18 (a) and (b) show the encoding as indicated by teachers T_1A and T_2B on their lesson plans respectively:

Correct expression of final answers to algebraic word problems by teacher (T_1A) (panel a) and by teacher (T_2B) (panel b) on the lesson plans.

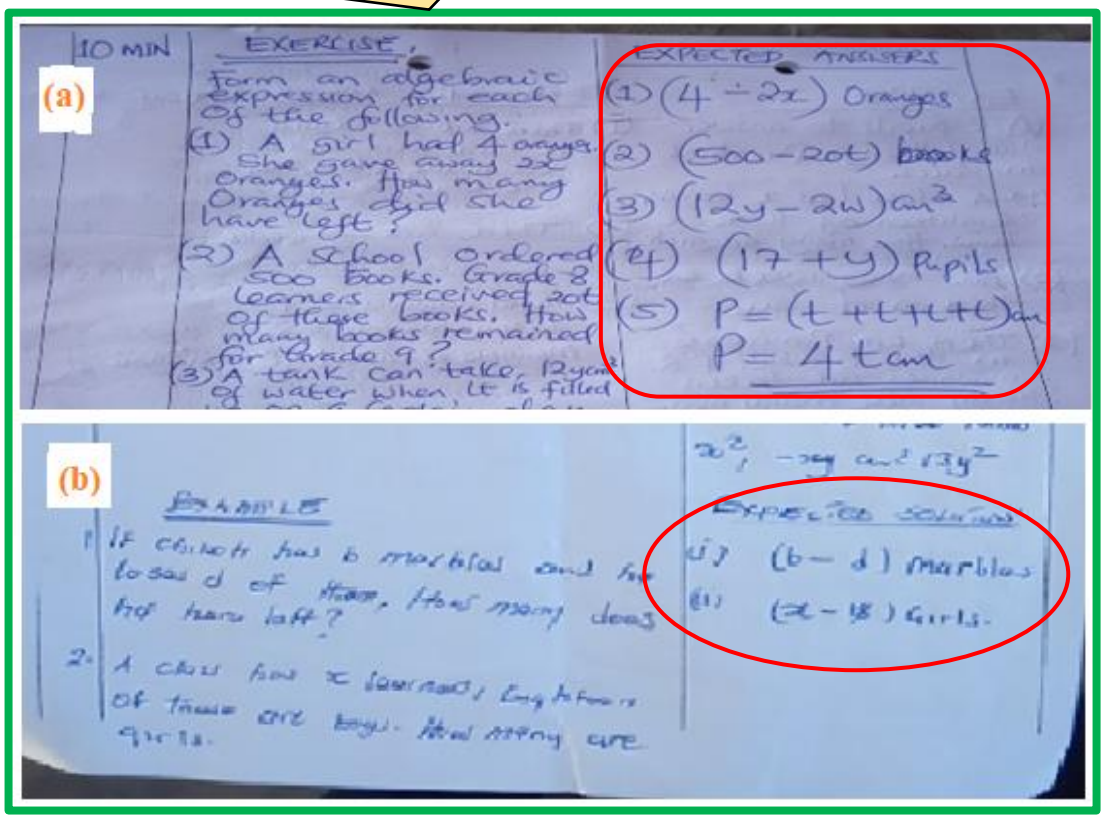

Figure 18: *Correct expression of final answers to algebraic word problems on the lesson plans by* T_1A *(panel a) and by* T_2B*(panel b).*

The following snapshots (Figure 19 (a) and (b)) show the correct encoding of some grade eight learners in the classes of these teachers who properly encoded solutions to algebraic word problems both on the board and on the lesson plans. In Figure 19 below,

panel (a) shows the learner's encoding in $T_1A's$ class, and panel (b) shows the learner's encoding of algebraic word problems in $T_2B's$ class. These learners encoded correctly the answers to the algebraic word problems because they saw it from their teachers, as already evidenced in Figures 17 and 18 respectively.

Correct encoding by the learner in $\boldsymbol{T_1A's}$ $(panel\ a)$ and $\boldsymbol{T_2B's}$ $(panel\ b)$class

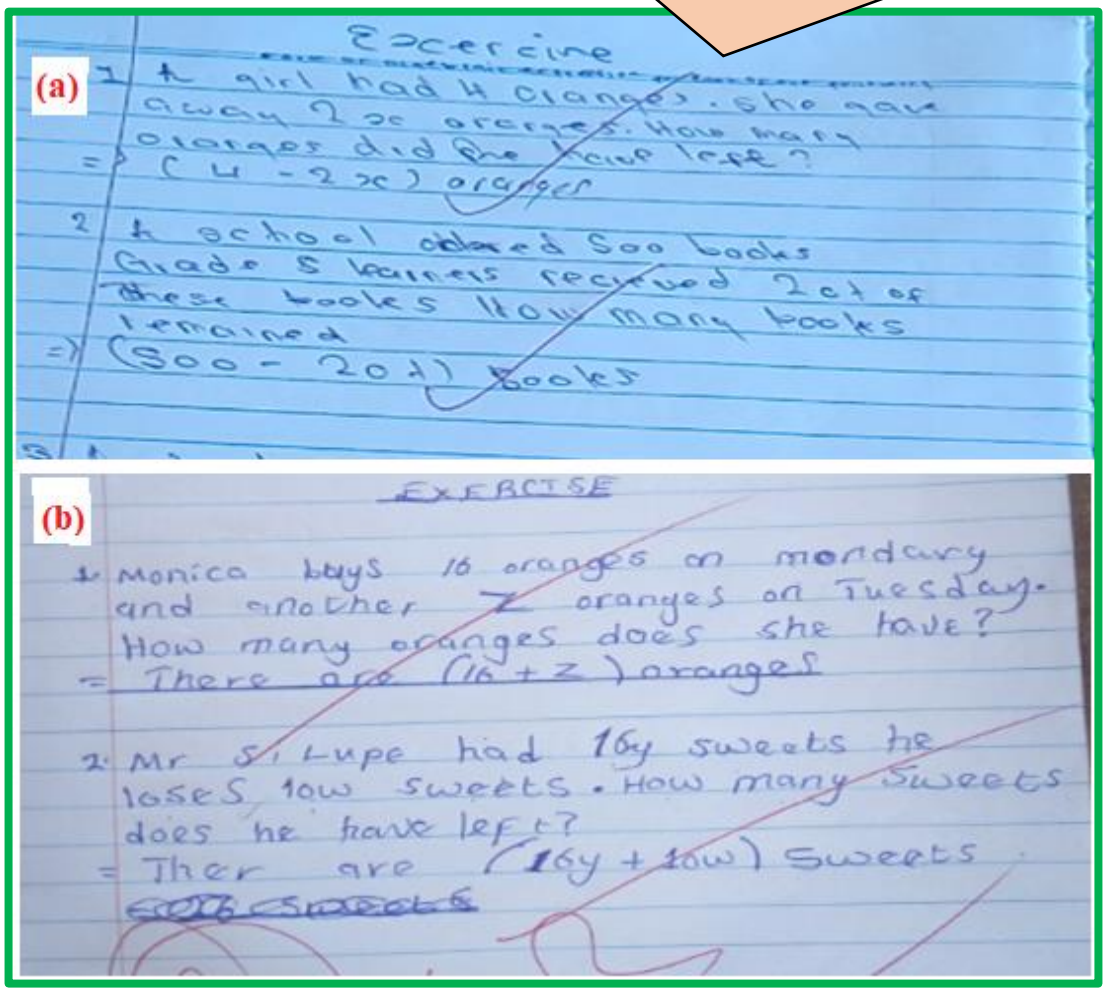

Figure 19: *Sample of learners' encoding of algebraic word problems in their books, in* $\boldsymbol{T_1A's}$ *class (panel a) and in* $\boldsymbol{T_2B's}$ *class (panel b).*

4.4.2.2 Partially correct in presenting final answers

The results from lesson observations revealed that teachers (1 out of 5) partially presented answers correctly. This partially correct way of presenting final answers to algebraic word problems was seen even on the lesson plans when conducting

document analysis. Figure 20 (a) and (b) show how T_2A presented final answers correctly on the board, and a mixture of correct and partially correct answers on the lesson plan respectively.

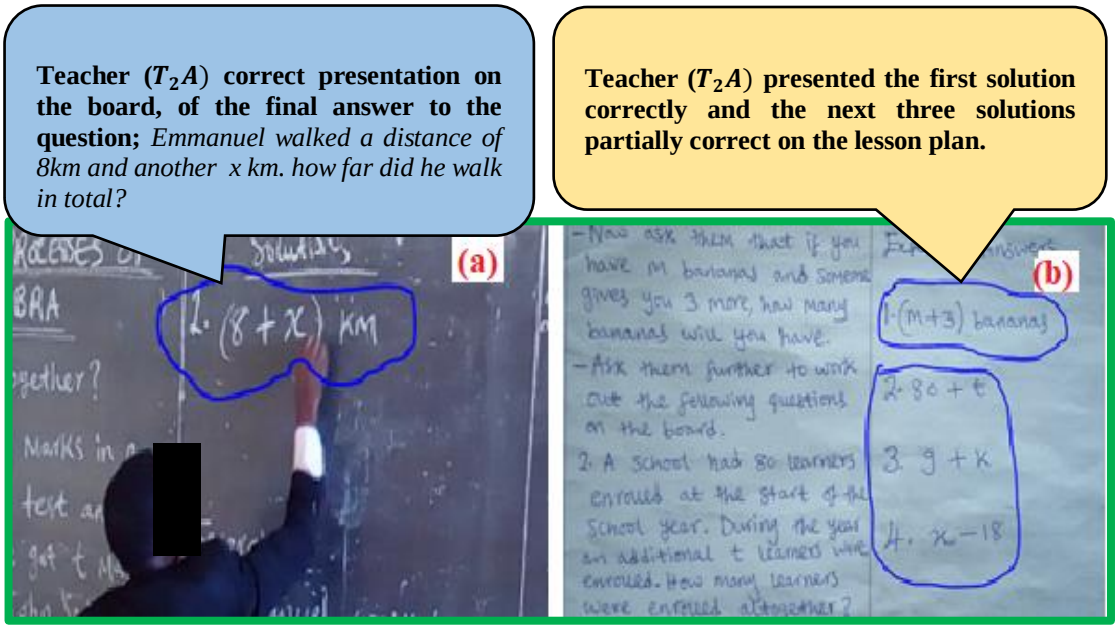

Figure 20: *Teacher (**T_2A**) correct presentation of final answer on the board (panel a) and partially correct presentation of final answers on the lesson plan (panel b).*

The teacher was not consistent in expressing final answers to the algebraic word problems. However, during interviews, this same teacher acknowledged the importance of being strict in writing and planning before teaching. The following are the excerpts by T_2A:

> *"Teachers need to be strict with planning and ensure that they present solutions to algebraic word problems systematically and consistently.....hence, it is part of my routine when teaching"* **(T_2A).**

FGDs revealed that teachers did not actually present solutions to algebraic word problems systematically and that left some learners in the confused state, not knowing which is the best way to answer and express final answers to the problems at hand. The following verbatim provides an example of learners' responses:

> *"Some teachers write answers differently and we get confused"* **(L_6A)**. *"Teachers should be showing us how to write the final answers….now sometimes they don't show us the correct way to write answers"* **(L_3B)**.

In Figure 20 (a), the discussions for the question were as evidenced under Figure 17.

4.4.2.3 Not strict with learners' encoding abilities

Results from lesson observations also revealed that teachers (2 out of 5) were not as strict when dealing with final answers of learners in algebraic word problems. Document analysis showed that teachers did not adequately plan how they were going to enhance learners' encoding abilities in algebraic word problems. Their lesson plans did not indicate how they intended to probe and enhance learners' encoding abilities despite showing some expected solutions to the problems (see Figure 18 for details, p.73). Even some of the expected solutions were not correctly written on the lesson plans. In some cases teachers paid attention to the spellings though not consistently but paid attention to symbols pretty well (see Figure 13 for details, p.66). Teachers accepted a number of solutions to the same question and marked them all correct despite some of them not being correctly done. For example, T_2A accepted the following four solutions: $(x-18)girls$, $x-18girls$, $xgirls-18girls$ and just $x-18$, as all correct answers to the question '*A class has x learners, eighteen of these are boys. How many are girls?'* The following snapshots (Figure 21) provide evidence of this:

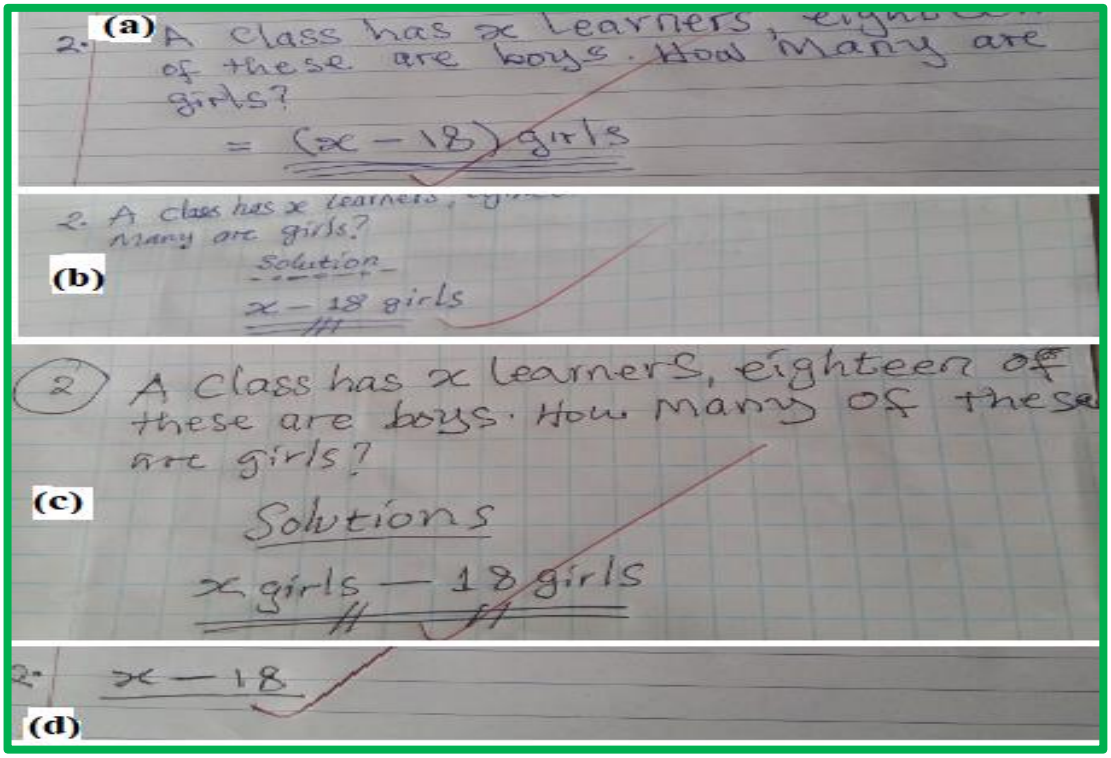

Figure 21: *Teacher* ***(T_2A)*** *marked four different answers to the same question as being correct in panels a, b, c and d.*

Teachers marked answers with symbol recognition in mind without considering the word recognition part as well. In some cases teachers did not pay attention to the units used in the question when looking at the final answers of the learners. Answers such as; $p = 2s + 2t$, $8 + 2xkm$, $8km + 2xkm$ were common and teachers endorsed them as correct when marking learners' books. For example, the following snapshots in form of Figure 22 and Figure 23, show the answers which T_2A and T_1B endorsed as correctly expressed when marking learners' exercise books respectively.

Teacher (T_2A) was not strict with learners' encoding abilities, hence marked correct the two learners who expressed answers to the same question differently. The first learner wrote ***17 pupils + y pupils***, and the second learner wrote ***17 + y mangoes***. The question was: *'There are 17 pupils in a classroom. If y more pupils come in, how many pupils are there altogether?'*

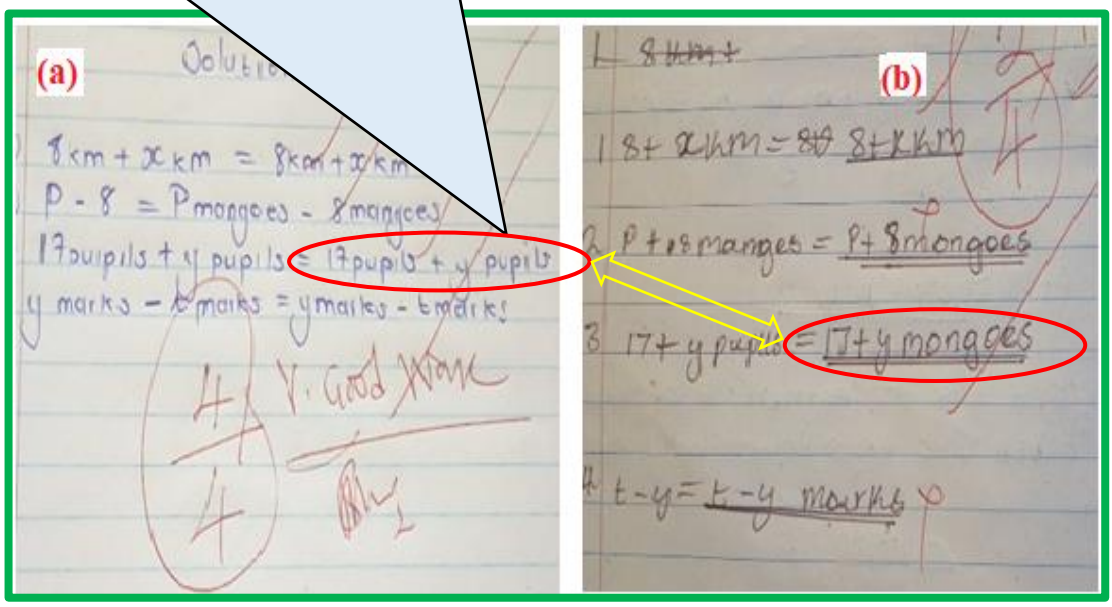

Figure 22: *Teacher (**T_2A**) not being strict with learners' encoding abilities marked two different answers to the same question (panels a and b).*

In both scripts, teacher (T_1B) was only strict with symbol recognition of learners and not word recognition. The questions were: *1. Chikoti has b marbles and he loses d of them, how many does he have left? 2. A grade 8 blue grade teacher counted 26 learners in her class and discovered that 3h learners were absent. What is the total number of learners?*

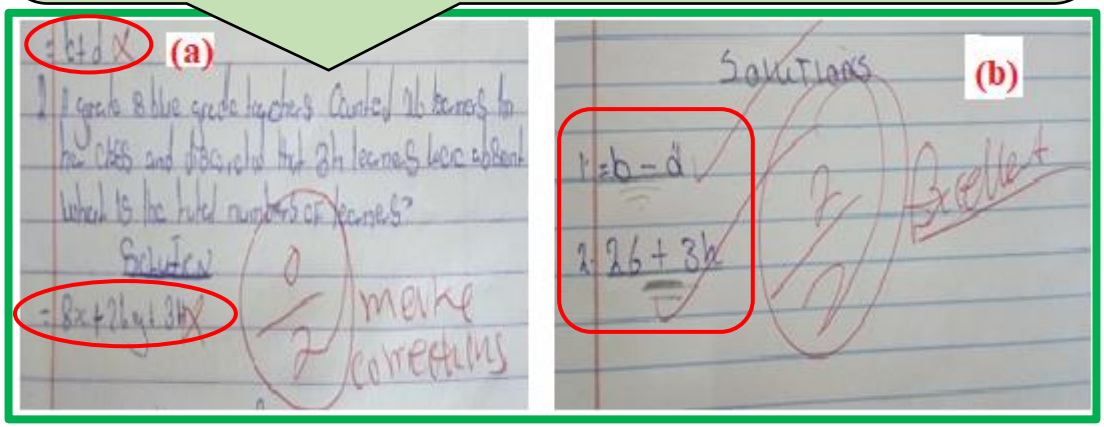

Figure 23: *Teacher* ***(T_1B)*** *being strict only with symbol recognition but not word recognition by learners in both panels (a and b).*

Results from interviews indicated that teachers are aware of the fact that some learners had challenges as they added together unlike terms and expressed the sum as a final answer to the given algebraic word problems. Teachers also noted that some learners were mixing letters and numbers. The following excerpts are a typical example of the teachers' responses when they were asked about what learners find challenging:

> *"Learners find it difficult to deal with like and unlike terms, sometimes they add together unlike terms"* ***(T_1C)***. *"Use of letters for numbers is problematic for learners as they prefer numbers to letters"* ***(T_1B)***.

When I asked them what they needed to do to ensure that learners become competent in solving algebraic word problems, the following were the extracts of some answers from T_1C and T_2B:

> *"We must allow learners to solve algebraic word problems on their own so that they can identify their own errors and our role is just to come in and help correct those errors"* ***(T_1C)***. *"I need to be the first one to express answers correctly so that learners can also follow suit"* ***(T_2B)***.

However, document analysis did not reveal how these teachers planned to deal with the encoding abilities of learners' in algebraic word problems, though some lesson

plans showed the encoding as implied in the way they wrote their final answers although not consistently done by some teachers as evidenced in Figure 6.

In FGDs, grade eight learners said that they had challenges in as far as expressing the final answers to algebraic word problem was concerned mainly because they missed it at understanding level. Many learners did not understand the signs and symbols as well as the variables and units and often mixed them when writing the final answers as evidenced in Figures 21, 22 and 23 respectively. When I asked learners in FGDs why they had challenges in answering algebraic word problems, the following excerpts from L_3A and L_4C are typical examples of their responses:

> *"I have difficulties in signs interpretation" (**L_3A**). "I face challenges with the use of letters for numbers" (**L_4C**).*

Having presented the findings of the study, I now present the summary of the findings for the research questions 1, 2 and 3. These findings are presented in Tables 3, 4 and 5 respectively. The chapter ends with the summary of the research findings with regards to the research objectives as presented in Table 6.

4.5. Summary of findings for research question 1

Table 3: *For research question 1 on how teachers engaged learners in reading algebraic word problems.*

Process	Description of the process	Findings
Reading of algebraic word problems	• Reading • Word recognition • Symbol recognition • Identifying key words	**Reading** • One out of five teachers gave learners chance to read algebraic word problems. • Four out of five teachers read algebraic word problems for the learners. • Four out of five teachers involved the whole class at once in chorus reading. **Word recognition** • Teachers did very little to probe learners' word recognition abilities. **Symbol recognition** • Teachers probed learners' symbol recognition ability very well. **Identifying key words** • Teachers did not underline key words in algebraic word problems. • Teachers did not ask learners to underline key words in algebraic word problems.

4.6. Summary of findings for research question 2.

Table 4: *For research question 2 on how teachers probed learners' understanding of algebraic word problems.*

Process	Description of the process	Findings
Understanding of algebraic word problems	• Giving learners chance to explain the meaning of algebraic word problems in their own words. • Literal understanding of algebraic word problems. • Specific terminology understanding of algebraic word problems. • Translation of algebraic word problems.	**Explaining in own words** • One out of five teachers gave learners chance to explain in their own words the meaning of algebraic word problems. • Four out of five teachers did not give learners chance to explain the meaning of algebraic word problems in their own word but explained for them. **Literal understanding** • One out of five teachers probed learners' general understanding of algebraic word problems. • Four out of five teachers did not probe learners' general understanding of algebraic word problems. **Specific terminology understanding** • Teachers did not do much in probing learners' understanding of specific terminologies in algebraic word problems. **Translation of questions** • Teachers did not adequately test if learners understood the algebraic word problems by asking them to translate the questions into the form which was sufficiently familiar for them to be able to select mathematical operations to be used to solve the problem.

4.7.Summary of findings for research question 3.

Table 5: *For research question 3 on what teachers did to enhance learners' process skills and encoding abilities in algebraic word problems.*

Process	Description of the process	Findings
Process skills and encoding abilities	Performing the actual processes required to gain an answer to algebraic word problems. (i) Arithmetic Random response Wrong operation Faulty algorithm Faulty computation No response (ii) spatial Logic (iii) words (iv) Symbols	**Performing actual processes** • Teachers (3 out of 5) allowed a number of learners to attempt answering questions on the board and then discussed the answers with them. • Teachers went round marking learners' books and soliciting learners' explanations of the processes where necessary. **Arithmetic, Spatial and Logic** • Teachers did very little to correct learners' errors noted in their processes concerning wrong operation, faulty algorithm, spatial and logic. **Words and symbols** • Teachers (3 out of 5) presented their final answers well on the board and lesson plans. • Teachers wrote answers on the board which were partially correct. • Teachers were not strict with learners' encoding abilities, hence they marked right a variety of solutions to the same questions even those that were wrongly expressed. • Teachers did not fully pay attention to how the words and symbols were expressed in the final answers.

4.8. Summary of the chapter

This chapter presented finding from the participants on the teachers' classroom practices in addressing learners' errors in algebraic word problems as presented in Table 6 below:

Table 6: *Summary of the chapter*

Objective 1: To examine how teachers engage learners in reading algebraic word problems in their classrooms.

Process	**Findings**
Reading (Word and symbol recognition)	• One out of five teachers engaged grade eight learners in reading algebraic word problems first. • Four out of five teachers read algebraic word problems for grade eight learners first. • Four out of five teachers involved grade eight learners in chorus reading after reading for them first.

Objective 2: To assess how teachers probe learners' understanding of algebraic word problems.

Ability	Findings
Comprehension	• One out of five teachers gave chance to grade eight learners to explain algebraic word problems in their own words first. • Four out of five teachers explained the meaning of word problems for grade eight learners first. • Two out of five teachers put grade eight learners in small groups to discuss algebraic word problems. • One teachers allowed grade eight learners to go direct into solving algebraic word problems without probing their understanding first.
Transformation	• One out of five teachers asked grade eight learners to translate algebraic word problems. • Four out of five teachers translated algebraic word problems for the learners without asking them to attempt first.

Objective 3: To determine what teachers do to enhance learners' process skills and encoding abilities in algebraic word problems.

Ability	Findings
Process skills	• One out of five teachers wrote down key information on the board when answering algebraic word problems. • Four out of five teachers did not write down key information when answering algebraic word problems. • Three out of five teachers allowed grade eight learners to attempt answering algebraic word problems on the board. • Two out of five teachers went round marking learners' exercises in algebraic word problems and asked learners to explain their processes where necessary.
Encoding (words and symbols)	• Two out of five teachers presented final answers to algebraic word problems on the board and lesson plans correctly. • One out of five teachers presented partially correct answers to algebraic word problems on the board and on the lesson plans. • Two out of five teachers were not strict with grade eight learners' encoding abilities in algebraic word problems. • Two out of five teachers paid more attention to symbols than words in the learners' final answers to algebraic word problems.

CHAPTER 5

DISCUSSION OF FINDINGS

5.1 Introduction

This chapter discusses the findings as presented in chapter four. Just like in the previous chapter, the discussion will be done in subsections. Section 5.2 discusses how teachers engaged grade eight learners in reading algebraic word problems. Sections 5.3 provides discussions on how teachers probed learners' understanding of algebraic word problems and Section 5.4 discusses what teachers did to enhance learners' process skills and encoding abilities in algebraic word problems in line with Newman's performance strategies. Finally, Section 5.5 summarises the chapter.

Research question 1: How do teachers engage learners in reading algebraic word problems?

5.2. Reading abilities (Word and Symbol recognition)

This sub-section provides the discussions of how teachers engaged grade eight learners in reading algebraic word problems to ensure that they acquired competences in both word and symbol recognition within Newman's (1977, 1983) first stage of performance strategies of solving word problems.

5.2.1 Giving chance to learners to read algebraic word problems

Results from lesson observations revealed that one (1) out of five (5) teachers gave chance to individual grade eight learners to read algebraic word problems first. The teacher called upon a good number of learners to read and reread the algebraic word problems aloud to the class before solving. The teacher who did that only read through the questions aloud to the class after a number of learners had read through (see subsection 4.2.1. for details, p.52). The teacher's practice was in line with Newman (1977) who stated that it is important to give learners chance to verbalize mathematical tasks. This simply means that teachers should give learners chance to read mathematical problems, especially word problems for them to ensure that learners exhibit competences in word and symbol recognition. In addition, Wertheimer (1959)

stated that reading a mathematical problem before solving is essential as it helps solvers to understand the problem at hand. In this case teachers need to treat learners as problem solvers and ensure that they are given enough time to read algebraic word problems first before thinking of solving. Reading word problems before solving helps to see if learners can decode key words and symbols used in the mathematical problem correctly (Kumar, 2018). This also helps in finding correct answers to the problem, as it is important to read the question first before solving.

Polya (1957) argued that it is foolish to answer the question which you have not read and you do not understand. Newman (1983) added that sometimes a learner cannot read the word such as 'divide' but does understand the process of division if expressed symbolically, hence the simple remediation step needed is for the teacher to teach the learner to recognize the word 'divide'. Therefore, it is important for teachers to engage learners in reading word problems in mathematics so that they can identify and correct learners' errors regarding word and symbol recognition.

5.2.2 Reading the questions for learners

Research findings from lesson observations revealed that four (4) out of five (5) teachers first read the questions for the class rather than engaging grade eight learners themselves in reading. After reading algebraic word problems, they only engaged learners in reading for the second time as a whole class with chorus reading aloud. However, interviews with these teachers revealed that they knew how important it was for grade eight learners to have reading abilities as it was crucial in solving algebraic word problems (Refer to subsection 4.2.2. for details, p.53). Document analysis also revealed that teachers did not plan how to engage learners in reading algebraic word problems. It is important for teachers to plan how to engage learners in reading algebraic word problems as it reminds them of what they are expected to do during practice. Teachers read questions for grade eight learners contrary to Halladay and Neumann (2012) who advised that teachers should expose learners to reading in mathematics so that they can make predictions, monitor understanding, determine importance and make connections.

Moreover, it is difficult for teachers who read word problems for learners to determine learners' prior knowledge and correct their errors. Such teachers may have challenges

in identifying learners' problems in word recognition errors, gaps in background knowledge, or confusion with figurative language (Halladay and Neumann, 2012), as it was seen in the classes for these teachers. Word problems are important because they show teachers and parents where learners are struggling to connect (Kumar, 2018). Hence, it is important for teachers to expose learners to reading algebraic word problems, as it may help them to discover learners' errors so that they can provide the necessary mediation to address them. It is only after the teacher establishes what learners can do unassisted and what they can do with assistance from either more capable peers or from the more knowledgeable others that they can provide the necessary mediation (Vygotsky, 1978).

Research question 2: How do teachers probe learners' understanding of algebraic word problems?

5.3 Understanding of algebraic word problems

This Section discusses how teachers probed grade eight learners' understanding of algebraic word problems, regarding how they engaged them in explaining the meanings of problems at hand within Newman's (1997) second and third stages of performance strategies of solving word problems.

5.3.1 Comprehension abilities (Literal and specific terminology understanding)

5.3.1.1 Giving learners chance to explain the meaning of the questions

This study established that one (1) out of five (5) teachers gave learners chance to explain the meaning of algebraic word problems at hand. It is important to give learners chance to explain the meaning of algebraic word problems, especially at a lower level like grade eight (the focus of this study) where learners should sharpen their comprehension abilities. In the classes where teachers give learners chance to explain the meaning of algebraic word problems, it becomes easy for the teachers to identify learners who demonstrate abilities in literal and specific terminologies understanding. This was seen in the class where learners were given chance to explain their understanding of algebraic word problems.

The above finding looks more of a solution to the findings by Adu, Assuah, and Addo (2015) who in their study examined learners' errors in solving linear equation word problems in Ghana using Newman Error Hierarchical Levels (HEAL) and established that about 75 percent and 86 percent of the learners made comprehension and transformation errors respectively which occurred during the translation of the statement to algebraic form. Therefore, by giving learners chance to explain the meaning of algebraic word problems in their own words, the errors which the above scholars noted at comprehension and transformation levels can be reduced. The teacher did the correct thing by asking learners to explain in their own words the meaning of algebraic word problems, because teaching in the ZPD requires teachers to teach through questioning and prompting of learners' ideas (Brodie, 2000).

Moreover, learners who expressed difficulties in literal and specific terminologies understanding were assisted in the process. This is a good classroom practice as it helps in understanding the learners' strengths and weaknesses thereby helping the teacher to come up with appropriate remediation to the errors noted. This finding is in line with Newman (1977) who argued that if the learner fails to answer word problems in mathematics correctly, the teacher must ask if the learner can read and understand the meaning of the question before launching on an elaborate remediation programme directed at the mathematical process involved. In addition, this finding is also in line with Mukunthan (2013) who established that learners' performance in word problems was weak and suggested that teachers should employ classroom practices which are aimed at identifying learners' error patterns so that they provide remedial measures.

Even though one (1) teacher gave learners chance to explain the meaning of algebraic word problems, that teacher did not encourage learners to identify and write down key words used in the algebraic word problems. Key words play an important role in helping problem solvers to explain the meaning of word problems in their own way (Montague, 2005). Polya (1957) stated that some mathematical problems contain unnecessary information, therefore, it is important that problem solvers take keen interest in identifying important information before solving. Kumar (2018) also noted that, word problems can be confusing because, unlike equations, they contain extra words, numbers, and descriptions that have seemingly no relevance to the question. Hence the importance of identifying key words in algebraic word problems cannot be

overemphasised as it aids in understanding of problems at hand leading to appropriate solving of problems.

5.3.1.2 Explaining the meaning of the questions to the learners

The study established that four (4) out of five (5) teachers did not give grade eight learners chance to explain the meaning of algebraic word problems in their own words, but rather explained for them. Teachers in this case took it for granted that all learners knew how to read and could understand mathematical problems expressed in words easily, when in fact not. Many learners in classes for these teachers did not show competences in understanding algebraic word problems as seen by the way they answered the questions in the exercises and group work activities given to them. Learners made all sorts of errors when answering algebraic word problems because teachers did not expose them to the culture of explaining the meaning of word problems before answering. Document analysis showed that teachers did not plan how to engage learners in explaining the meaning of algebraic word problems. Some teachers only indicated that learners were simply going to listen to their explanations. However, learners need to be treated as active participants in their own learning so that they can be helped to redefine their situation definitions in order for them to make the transition from actual developmental level to potential developmental level in the ZPD (Wertsch, 1979). Hence, planning by teachers on how to engage learners in their own learning is crucial if learners are to develop competences in the topic.

The finding is closely related to that of Salihu (2017) who investigated grade 11 learners' errors when solving Algebraic word problems in Gauteng, South Africa and found out that 90 percent of learners demonstrated unfitness due to poor linguistic proficiency, failure to understand algebraic terminologies, and the gap between arithmetic and algebra. This is related to the above finding because teachers who do not ask learners to explain algebraic word problems are not likely to develop learners' competences in understanding algebraic terminologies.

Moreover, no matter how well the teachers explained the meaning of algebraic word problems to the learners, they still made a lot of errors when doing the exercises and the group work activities which were given to them. This is so because the teachers' classroom practices did not allow learners to master the concepts. This clearly

demonstrated that it is crucial for teachers to let learners take a leading role in their own learning for them to master the concepts of algebraic word problems.

This is in line with Osterholm (2006) who revealed that although the University students had studied more mathematics courses than the upper secondary learners, there was only a small and insignificant difference between these groups regarding reading, comprehension of mathematical text with symbols. The findings suggested that there is a need for more explicit teaching of reading comprehension for texts including symbols. Moreover, Holohan (2018) discouraged teachers from jumping in too quickly to help or assume learners know how to tackle the problem but to allow them explore the problems on their own first. Sociocultural theory of development also encourages teachers to give learners opportunities to explain mathematical concepts first, so that their prior knowledge can be checked and used in the learning of new concepts (Wertsch, 1979). Hence, teaching of algebraic word problems should aim at developing learners' competences.

5.3.1.3 Putting learners in small groups to discuss the questions

Results from lesson observations also revealed that two (2) out of five (5) teachers put learners in small groups where they discussed algebraic word problems which were presented to them by the teacher (see Figure 9 (a) and (b) for details, p.59). Some groups were randomly sampled to present their discussions and solutions (see Figure 10 (a) and (b) for details, p.60). In Figure 11 on page 61, the learner made an error by replacing a variable *k* with a number 4. This error is common among learners. They like replacing a variable for a number. For them leaving the answer as *(6+k)* is incomplete. This resonates with Nalube (2014) who observed that conjoining error was common among learners as they often replaced a variable by any number for them to add and subtract algebraic expressions easily.

Giving learners chance to explain was a great opportunity presented to grade eight learners to bring out their reasoning concerning the tasks at hand. This is good because even those learners who are shy to speak out to the whole class are likely to speak if they are discussing in a small group as it was seen in these classes. This is in line with sociocultural theory of development in the sense that knowledge was socially constructed in the groups. Slow learners learnt from collaborating with more capable

peers in the process of discussing. In that case, learners are likely to progress intellectually in their ZPD (Vygotsky, 1978). Hence, it is important for teachers to put learners in small groups to discuss algebraic word problems since some of them learn well from their peers (Brodie, 2000). Small group discussion is one way of bringing spontaneous concepts into the classroom. These spontaneous concepts are necessary in the formation of scientific and true concepts (Wertsch, 1979). Therefore, it is important to put learners in small groups to discuss problems such as algebraic word problems as it helps them to develop a deeper understanding.

Moreover, this resonates with Mukunthan (2013) who established that more than three quarters of the learners made errors before performing any calculations, because they had challenges in reading, comprehension, and transformation. Koji, Mulenga, and Mukuka (2016) also established that lack of conceptual, procedural and strategic knowledge by learners was one of the challenges faced in answering algebraic linear equations. Small groups provide opportunities for learners to show some understanding before they embark on solving mathematical problems. Furthermore, this finding is in line with Even and Tirosh (2002) who encouraged a classroom culture where learners are allowed to make conjectures, explain their reasoning, validate their assertions, discuss and question their own thinking and the thinking of others, and argue about what is mathematically true. Hence, putting learners in groups to discuss algebraic word problems may provide a way for them to acquire competences in comprehension, conceptual, and procedural knowledge.

5.3.1.4 Allowing learners to go direct into solving without showing some understanding

Lesson observations also revealed that one (1) out of five (5) teachers allowed learners to go direct into solving algebraic word problems without showing some understanding of the questions at hand through verbal explanations. In this case, learners may learn algebraic word problems without some understanding. Lambdin (2009) noted that learners who have learnt mathematics without understanding are often successful only in solving problems similar to those they have already seen, and are unable to see how mathematical ideas are related or useful. These learners often see the subject itself in a negative light, viewing it as arbitrary and mysterious – a

subject which only geniuses can master. Teaching and learning in the social setting as advocated by sociocultural theory of development requires teachers to probe learners' understanding first so that their prior knowledge is established. The teacher can then use their prior knowledge to help them understand concepts at hand (Vygotsky, 1978). Hence, teachers need to teach learners to develop the skills of communicating effectively with the teacher and other learners, which entails articulating situation definitions, negotiating intersubjectively and working with each other's ZPD's (Brodie, 2000). It is therefore, imperative for teachers to probe learners' reasoning about word problems at hand before asking them to provide answers or call upon them to write answers on the board, as this can help them understand algebraic word problems.

However, interviews with the same teacher revealed that he knew how important it was to probe learners' reasoning of algebraic word problems before asking them to give out their attempts on the given problems, though he did not put that into practice. It does not suffice to call upon learners to solve algebraic word problems on the board when the teacher has not probed their understanding through verbal explanations in the first place. As it has been noted by many scholars, it is difficult for learners who have not demonstrated some understanding to solve algebraic word problems. For example Ural (2016) observed that failure to understand the main idea of the problem is the main reason why learners fail to solve word problems such as algebraic word problems. In FGDs, learners also indicated that they preferred being asked to verbalise their understanding of algebraic word problems at hand before asking them to solve. Therefore, teachers must endeavour to probe learners' understanding before asking them to solve algebraic word problems.

5.3.2 Transformation abilities

5.3.2.1 Asking learners to translate algebraic word problems

Lesson observations revealed that one (1) out of five (5) teachers asked learners to translate algebraic word problems into the forms familiar to them for them to be able to solve the problems easily. This is important as it helps teachers to identify areas where learners may have challenges. The teacher who did this identified learners' errors and provided the necessary mediation there and then. Even during interviews,

this teacher stressed the prominence of probing learners' translations of algebraic word problems, and the importance of this aspect cannot be overemphasised. This is in line with Mukunthan (2013), and Newman (1977) who established that most of the learners made errors on the stages of reading, comprehension, and transformation. Hence, teaching learners in such a manner is likely to enhance their competences in comprehension and transformation. By translating algebraic word problems, learners are likely to have some understanding of the concepts which can lead to having confidence in the topic and subject at large. When learners appreciate the underlying structures of mathematics, they see it as a reasonable, approachable-subject; as a result their self-confidence with mathematics rises and they are generally more willing to tackle challenging problems (Lambdin, 2009). However, document analysis showed that there was no planning of how to engage learners in transformation of algebraic word problems by all the teachers. Teachers should show in their planning how they intend to probe learners' transformation abilities, since proper planning prevents poor performance. Moreover, social constructivism sees learners as coming to the learning with some understanding, which may be correct or wrong. This is a more reason why teachers need to probe learners' transformation abilities as it can help them to identify learners' errors which are necessary for learning in the ZPD and provide appropriate mediation to address them (Vygotsky, 1978). Hence, it is important for teachers to probe learners' transformation of algebraic word problems as it may help them to identify and correct learners' errors.

5.3.2.2 Translating algebraic word problems for the learners

Results from lesson observations also revealed that four (4) out of five (5) teachers translated algebraic word problems for the grade eight learners without asking them to state their reasoning of the problems presented to them first. This is in contrast with Newman (1977) who advised that learners should be given time to state word problems in forms familiar to them before answering, as it helps them to develop some understanding of the language of mathematics. Moreover, translating algebraic word problems for the learners does not necessarily mean that learners would grasp the concepts but it may rather discourage them to work hard as they would entirely depend on the teacher, hence not putting in much effort in their studies. It is important for teachers to ask learners to translate information in their own way as it helps in

determining learners' prior knowledge and also errors and gaps which they have concerning the problem at hand (Halladay and Neumann, 2012). Moreover, learning in the ZPD requires teachers to allow learners to bring out what they already know concerning the problems at hand. That is the only way the teachers can determine what learners can do on their own and the errors they make as they attempt to explain. This enables the teacher to find appropriate ways of mediating the errors observed and help learners to understand the concepts (Vygotsky, 1978). Teachers did not put this aspect in their planning too as shown by document analysis. FDGs however, revealed that learners were eager to learn through being probed. Therefore, giving learners chance to translate algebraic word problems by teachers is important as it provides the needed feedback which helps the teachers to modify classroom instructions in order for learners' errors to be addressed. This can then lead to conceptual understanding of algebraic word problems.

Research question 3: What do teachers do to enhance learners' process skills and encoding abilities in algebraic word problems?

5.4 Process skills and encoding abilities in algebraic word problems

This section discusses the findings of research question 3 of this study, which aimed at exploring what teachers do to enhance learners' process skills and encoding abilities in algebraic word problems.

5.4.1 Process skills

5.4.1.1 Writing down key information from word problems

It was seen from lesson observations that one (1) out of five (5) teachers when answering examples on the board did extract key information from the algebraic word problems before writing down the final answers (see Figure 12 for details, p.65). This teacher did this even for the questions that were viewed to be straight forward. This is important as it aids learners' understanding of key information which could help them in coming up with correct solutions to word problems in general, and in particular, algebraic word problems. This finding is in line with Boonen, Koning, and Jolles (2016) who investigated the difficulties learners faced in language that is used in mathematical word problems and established that even successful word problem

solvers had a low performance due to issues of language used in problems. Emphasising or highlighting key information is designed to help learners restate the mathematical problems in their own words, thereby strengthening their understanding of word problems (Cunningham and MacGregor, 2014; Polya, 1957).

For example, problems such as, *'A class has x learners, 18 of these are boys. How many are girls?'* Teachers need to guide learners in writing key information, such as: *Total number of learners = x; Boys = 18; Girls =? Hence, Number of girls = (x – 18), or* simply *(x – 18) girls*. Learners who learn in this way are more likely to be systematic in their approach when answering algebraic word problems than those who do not. A major goal of school mathematics instructional programs must be to support learners in becoming autonomous learners, to understand what they learn, must enact for themselves verbs that permeate the mathematics curriculum: examine, represent, transform, solve, apply, prove, communicate, and many others (Lambdin, 2009). Moreover, the third principle of social constructivism derives from the sociology of knowledge, and acknowledges that reality is constructed intersubjectively, that is it is socially negotiated between significant others who are able to share meanings and social perspectives of a common lifeworld (Berger and Luckmann, 1966). In this case, teachers being the more significant others should communicate the processes of answering algebraic word problems to learners correctly and consistently in order for learners to gain competences. Writing key information from algebraic word problems first, before solving can by no doubt enhance learners' understanding of algebraic word problems.

5.4.1.2 Not writing down key information from word problems

Four (4) out of five (5) teachers as observed from their lessons did not write down key information from the algebraic word problems before solving (see Figure 13 (a) and (b) for details, p.66). Such teachers did not even encourage learners to do so. They only expressed the final answers to the questions because they thought that most of the questions were straight forward. Such classroom practices do not help in identification and correction of learners' errors. It is more likely that learners can struggle to find the final answers from the many words given in the problems if the teachers do not inculcate the culture of extracting key information from the given words in algebraic

word problems. Document analysis also revealed that these teachers did not pay attention to extraction of key information from algebraic word problems before solving.

For example, some teachers expressed the answer as '$b - d$' to the question, *'Chikoti has b marbles and he loses d of them, how many does he have left?'* For such questions, it is important for the teacher to encourage learners to show the processes as follows: *'Total number of marbles = b; marbles lost = d; marbles left = (b – d) or (b – d) marbles.'* If that is done consistently, learners are likely to develop the competences in the process skills necessary to answer word problems. Under appropriate instructional conditions, word problems not only can be negotiated successfully by the learners, but they can also be the vehicle to promote greater mathematical reasoning and achievement (Monroe, Panchyshyn, and Bahr, 2006). Moreover, in the social setting, learners learn from the adults (Vygotsky, 1978). It is for this reason that teachers should consistently teach a culture of being systematic to leaners. Through such practices, teachers can identify and correct learners' errors in algebraic word problems. Therefore, teachers need to teach in a systematic way to achieve that.

5.4.1.3 Allowing learners to write answers on the board

It was also observed from lesson observations that three (3) out of five (5) teachers called upon a number of grade eight learners to attempt answering algebraic word problems on the board. Teachers allowed learners to write their attempts on the board, whether correct or wrong and discussed all the attempts afterwards (see Figures 14 (a) and (b) for details, p.67). For example, in the discussions for Figure 14, some learners made an error by considering x to be the number of girls. These learners were unable to see that $(x - 18)$ is the number of girls because to them the letter x represents the unknown and since the number of girls was unknown they took it that x was equal to the number of girls. This was a misrepresentation of the variable x. However, through discussion this error was corrected. This is a good classroom practice as it allows teachers to evaluate learners' abilities in the process skills. Although teachers did not ask learners to explain the processes needed to gain a solution, by allowing them to write on the board shows that they still probed learners' process skills. This is in line with Newman (1983) who stated that if the learner cannot explain the meaning of the

question in his own words and explain the process but is able to show in writing, it is assumed that comprehension and process skills are implied even though not verbalised. Therefore, for the learners who ably wrote the correct solutions on the board, it was clear that they had abilities in comprehension and process skills. Lambdin (2009) added that learners learn more and better when they are helped to take control of their own learning.

Moreover, in the social construction of knowledge, teachers are encouraged to give learners chance to show their reasoning either by writing or through explaining. This leads to learners bringing out their spontaneous concepts to the learning. Discussing their attempts enables them to refine their spontaneous concepts in algebraic word problems and that helps learners to develop conceptual understanding through collaborating with other learners and the teacher (Vygotsky, 1978).

5.4.1.4 Going round marking and asking learners to explain their processes

Lesson observations also revealed that even if some teachers did not ask learners to explain the processes required to find the solutions when teaching algebraic word problems, they did so when going round marking learners' exercise books. These teachers asked learners who had challenges to explain their thinking with regards to their process skills. (see Figures 15 (a) and (b) for details, p.68). In Figure 15 (a), the learner made an error by subtracting unlike terms *4 – 2x* to get *2x oranges* as the answer. The learner in this case subtracted 2 from 4 to get 2 and then combined it with *x* to get *2x*. This is again a conjoining error as noted by Nalube (2014) that learners like to bring together unlike terms to get one thing. Therefore, teachers need to pay attention to this error as they teach algebraic word problems.

Although teachers went round marking and correcting learners' errors, they did not care about learners showing their processes in their exercise books when answering the questions (see Figure 15 (b) for details, p.68). In Figure 15 (b) for example, the learner's thinking was that in algebra every number should be accompanied by a letter, hence assigning *x* to 8 and *y* to 26. For this learner, 3*h* was complete since there was a variable already. The number 8 in the question represented a grade. For the learner to have added it to find the total number of learners in class as *8x + 26y +3h* is a clear indication that the context of the question was not well understood. Therefore, teachers

should ensure that they write key information from the word problems before writing the final answer. The practice of not caring about learners' process skills may not help in enhancing knowledge acquisition in the topic as it is not a good practice. This resonates with the findings of Koji, Mulenga, and Mukuka (2016) who established that inappropriate approaches and methods used in the teaching of algebraic linear equations was one of the reasons why learners faced challenges in the topic. Hence there is need for teachers to ensure that classroom practices do help learners to overcome any challenges they might be facing, especially in word problems in general, and specifically algebraic word problems.

However, interviews with these teachers indicated that they knew how important it was to pay attention to learners' processes of answering algebraic word problems even though they did not take that into practice. In FGDs, learners eluded to the fact that teachers needed to show them the processes of solving algebraic word problems. Document analysis also showed that teachers did not care so much on the processes of answering algebraic word problems but on the final answers. This was seen by the way they expressed solutions on the lesson plans (see Figure 18 (a) and (b) for details, p.73). Thus, it is important that teachers teach algebraic word problems systematically, and paying attention to the processes of solving is part of the systematic presentation of work.

The learner's zone of proximal development is assessed through interaction or collaboration with a learner because it provides an opportunity for imitation, which is the way for identifying maturing psychological functions that are still inadequate for independent performance (Vygotsky, 1998). Hence, by asking learners to explain their processes of solving algebraic word problems, teachers can mediate maturing psychological functions of learners thereby enhancing their problem solving abilities and eventually help them become independent problem solvers. As learners continue to practice, they can do certain tasks independently in activities which were previously executed with assistance. The changes the learners gain in understanding help them to find a way of attempting the problems which they were unable to solve even with assistance (Siyepu, 2013).

5.4.2 Encoding abilities (Word and symbol recognition)

5.4.2.1 Presenting final answers correctly

Results from lesson observations revealed that two (2) out of five (5) teachers presented the final answers to word problems on the board very well. These teachers paid attention to both symbols and words in the way they expressed their final answers (see Figures 17 and 18 for details, p. 72 - 73). These teachers also presented solutions to algebraic word problems on their lesson plans correctly as seen during document analysis (see Figure 18 for details, p. 73). It is imperative for teachers to express final answers to algebraic word problems correctly as this practice is likely to be transferred to the learners. This can result in the reduction of errors by learners, leading to them having conceptual understanding of algebraic word problems. This is in line with Sammons (2018) who observed that, for all learners, including those who are attempting to put their ideas down on paper for the first time, those who are refining their general writing skills, and those who are learning to express their mathematical ideas in writing, some of the productive learning experiences result from observing teachers as they model the writing process and speak aloud about their thinking as they do. According to O'Connell and Croskey, 2018, p.60), "Writing is a process through which we recognise our ideas, develop conjectures, and gain insights. It is not content itself, but a means of exploring and expressing content." Therefore, teachers must be careful in the way they write and express their final answers knowing that learners are going to learn everything they do.

Sociocultural theory on the other hand stresses that the most beneficial learning environment is one where a dynamic interaction between teachers, learners and tasks provides an opportunity for learners to create their own understanding during interaction with others (Atkinson, Derry, Renkl and Wortham, 2000). Atherton (2005) emphasised that in a sociocultural classroom, learners are active creators of meanings and the role of the teacher is to guide learners to gain meaningful understanding of the learning material. Therefore, the teacher when teaching algebraic word problems should guide the learners by solving problems systematically and by presenting final answers correctly and consistently in order for learners to have conceptual understanding.

5.4.2.2 Partially correct in presenting final answers

It was observed that one (1) out of five (5) teachers was not consistent in the presentation of the final answers to algebraic word problems. This teacher could present final answers in a mixed way such that some answers were properly presented while others were only partially correctly done (see Figure 20 (a) and (b) for details, p.75). Presentation of final answers in such a manner by teachers may have effects on the encoding abilities of learners who might think that final solutions to algebraic word problems can be presented in any way the solver likes. For example, for these two questions: *(i) If you have m bananas and someone gives you 3 more, how many bananas will you have? (ii) A class has x learners, eighteen of these are boys. How many are girls?* The teacher correctly expressed the answer to the first question as *'(m + 3) bananas'* but expressed the answer to the second question partially correctly as *'x – 18'* instead of including the word part as well, so that the answer is completely correct and reads as (x – 18) girls (see Figure 20 (b) for details, p.75). Leaners should know that ($x - 18$) represents a single unknown number of girls in this case by using parenthesis and showing the words which describe that number, that is 'girls' in this case. However, during interviews teachers stressed the importance of being consistent when answering algebraic word problems. Document analysis also showed inconsistence in presentation of final answers by some teachers.

This finding resonates with the observation by Sammons (2018) who established that teaching learners how to write about mathematics is a challenge for many teachers, just as there has been little clarity about the kinds of mathematical writing learners should do, minimal guidance has been provided about how to do it. Wertsch (1985, p. 67) stated that ZPD "is to deal with two practical problems in the learning situation: the assessment of learners' intellectual abilities and the evaluation of instructional practices". Learning activities challenge learners' thinking within the learning process. Hence, teachers should ensure that learning activities are well written and presented in order for learners to gain conceptual understanding. Moreover, the ZPD requires that teachers should be more knowledgeable in order to assist learners to gain understanding of what they cannot do without assistance (Siyepu, 2013). This knowledge also includes how they express final answers to algebraic word problems. Therefore, teachers need to be consistent in expressing final answers as it is a way of

teaching learners writing skills in mathematics which can in turn help them in reducing on committing encoding errors in algebraic word problems.

5.4.2.3 Not strict with learners' encoding abilities

Lesson observations also revealed that two (2) out of five (5) teachers were not strict with learners' encoding abilities. This was seen by looking at the solutions to algebraic word problems which they marked as being correct in the learners' books. It was clearly seen that these teachers paid attention only to symbol recognition by learners than paying attention to both word and symbol recognition (see Figures 21 and 22 for details on pages 76 and 77 respectively).

For example, the teacher who marked answers: *(x – 18) girls; x – 18 girls; x girls – 18 girls, and x – 18,* as all correct answers to the question, *'A class has x learners, eighteen of these are boys. How many are girls?'* (see Figure 21 for details, p. 76). The teacher marked correct four different answers to this question when in fact only the first one was completely correct and the last one was partially correct. The second and third solutions were wrongly done by the learners. The teacher marked these solutions correct, contrary to Newman (1983) who advised that if a learner fails to produce an acceptable written answer, even though he does know the correct answer, he may be marked as being incorrect. This guidance is aimed at encouraging teachers to be strict when dealing with learners' solutions so that their errors are lessened at the encoding stage of algebraic word problem solving.

Moreover, this kind of teachers' practices may lead to learners committing encoding errors when solving word problems like the finding of the study by Adu, Assuah, and Addo (2015) which established that 86 percent of the learners made encoding errors which occurred at the final stages of the work. Hence, it is imperative for that teachers to ensure that learners develop competences in encoding by being strict with the way they express final answers when teaching and marking in class. Crespo (2000) argued that, knowing what to look for and what to do with learners' mathematical utterances, whether in written or in oral form is not a trivial task for any teacher.

However, during interviews, these teachers stressed the importance of being strict with learners work all the times. This is correct but unfortunately they did not put that into

practice when teaching algebraic word problems. Learners' in FGDs also wondered why sometimes teachers marked totally different solutions to the same questions, adding that the situation made a lot of them get confused about algebraic word problems.

Vygotsky's ZPD, which explains how to advance learners' learning process is reinforced by Wertsch (1985, p. 60-61) who asserted that: Any function in the learner's cultural development appears twice, or on two planes. Teachers should be aware of the fact that learners learn from what they see them doing and also from what they observe other people do in society. Therefore, teachers should examine both oral and written responses to tasks of learners so as to ensure their conceptual growth in expressing their final answers to word problems, particularly algebraic word problems. Teachers should also express the final answers correctly and consistently so that learners can learn from them, as they viewed as role models in the classrooms.

5.5 Summary of the chapter

This chapter discussed the findings of the study on teachers' classroom practices in addressing learners' errors in algebraic word problems. The findings revealed that although teachers involved grade eight learners in their own learning in some way, they did not adequately probe learners' reading, comprehension, transformation, process skills, and encoding abilities for them to thoroughly identify and correct learners' errors when teaching algebraic word problems. I now present conclusion and recommendations in the next chapter.

CHAPTER 6

CONCLUSION AND RECOMMENDATIONS

6.1 Introduction

This chapter concludes the study and highlights the major contribution the study has made to research (Section 6.3). Thereafter, the recommendations (Section 6.4) are passed in order to improve teachers' classroom practices in addressing learners' errors in algebraic word problems. Because these errors by learners in algebra are transmitted to other topics in mathematics thereby contributing greatly to poor performance in mathematics as reported by the Examinations Council of Zambia 2017 and 2018. The chapter ends by suggesting further research in mathematics education.

6.2. Conclusion

The study explored teachers' classroom practices in addressing learners' errors in algebraic word problems by looking at how teachers engaged grade eight learners in reading; how they probed grade eight learners' understanding; and what they did to enhance grade eight learners' process skills and encoding abilities in algebraic word problems.

It was established that although some teachers gave grade eight learners chance to read algebraic word problems first, some of them chose to read for the learners without giving them chance first. Teachers who read algebraic word problems for the learners, only involved them with chorus reading after they had read for them first. Some teachers gave grade eight learners chance to explain in their own words the meaning of algebraic word problems at hand, while others explained the meaning of algebraic word problems for them. Some teachers put learners in small groups where they discussed algebraic word problems assigned to them. Some teachers asked learners to translate algebraic word problems in their familiar ways knowing that learners have idiosyncratic ways of reasoning, while others did not ask learners to do so, but did it for them. Teachers in some cases were seen allowing learners to attempt solving algebraic word problems on the board. Learners who made some errors in solving were assisted during the discussions of attempted solutions. Teachers also went round

marking and asking learners to clarify their processes and answers. Moreover, some teachers extracted key information from algebraic word problems when answering questions, and expressed final answers correctly, while others did not do so.

Generally, it was concluded that, although teachers engaged grade eight learners in reading, explaining, and solving algebraic word problems, they did not adequately follow the five steps of solving word problems as outlined by Newman. Teachers were inconsistent in as far as probing learners' understanding of algebraic word problems was concerned. Moreover, teachers did little to enhance learners' process skills and encoding abilities as even themselves were not consistent in the way they expressed the final answers when solving algebraic word problems. Therefore, the study established that teachers did not do enough to discover and correct learners' errors in algebraic word problems.

6.3 Contribution to the field

Worldwide, this study has also made an academic contribution to the body of knowledge regarding research in mathematics and teachers' classroom practices, learners' errors and Algebra education. Newman's (1977, 1983) performance strategies of solving word problems in mathematics and Vygotsky's (1978) Sociocultural theory of development, particularly the ZPD helped me to emphasise appropriate analysis of data on how teachers engaged grade eight learners in reading; how they probed grade eight learners' understanding (Comprehension and transformation), how they enhanced grade eight learners' process skills and encoding abilities. These concepts were applied in some studies before, but focused on analysing learners' errors of written mathematical tasks and not using them to explore teachers' classroom practices in addressing learners' errors. In view of the findings of this study, I made the following recommendations:

6.4 Recommendations

The following recommendations were made in the light of policy, practice, and further research.

6.4.1 Policy

1. There is need for MOGE through CDC to incorporate and emphasise Newman's performance strategies of solving word problems in the curriculum as a method of teaching word problems.

2. MOGE should intensify CPDs for teachers of mathematics in order to improve their pedagogical and social skills in algebraic word problems.

6.4.2 Practice

1. Both in-service and pre-service teachers need to undergo staff development so that they can handle algebraic word problems in a way that is aimed at identifying and addressing learners' errors.

2. Schools to intensify School Based Continuing Professional Development (SBCPD) so that teachers of mathematics can share good classroom practices regarding teaching of algebraic word problems.

3. Teachers should ensure that learners take a leading role in their own learning of algebraic word problems so that they can establish what learners can do unassisted and what they can only do with assistance, so that they can intervene by providing the necessary mediation.

4. Teachers should be strict with learners' processes and final answers in algebraic word problems. If that is consistently done, learners can reduce on making process and encoding errors.

5. Teachers should improve in planning on how they intend to engage learners in learning algebraic word problems. Planning serves as a reminder when it comes to practice.

6.4.3 Further research

1. There is need to examine the relationship between teachers' classroom practices and learners' performance in algebraic word problems.

2. There is need to examine learners' views on teachers' classroom practices in algebraic word problems.

3. There is need to compare grade eight learners' performance in word problems and similar arithmetic type of questions.

4. There is need to investigate learners' views on their classroom participation in mathematics lessons and in algebraic word problems.

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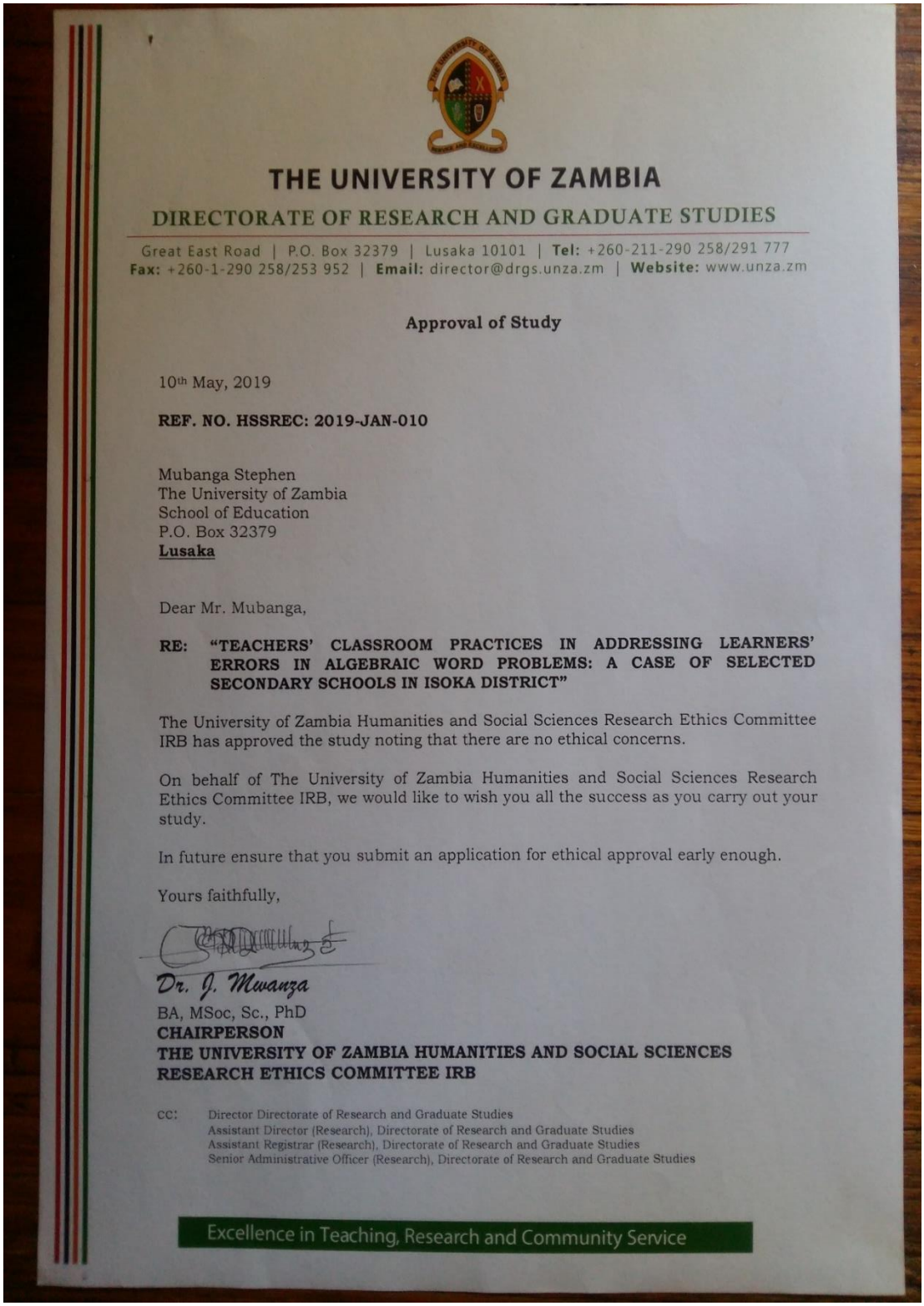
APPENDICES

APPENDIX A

Clearance letter for data collection from The University of Zambia Ethical Committee



THE UNIVERSITY OF ZAMBIA

DIRECTORATE OF RESEARCH AND GRADUATE STUDIES

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Approval of Study

10th May, 2019

REF. NO. HSSREC: 2019-JAN-010

Mubanga Stephen
The University of Zambia
School of Education
P.O. Box 32379
Lusaka

Dear Mr. Mubanga,

RE: "TEACHERS' CLASSROOM PRACTICES IN ADDRESSING LEARNERS' ERRORS IN ALGEBRAIC WORD PROBLEMS: A CASE OF SELECTED SECONDARY SCHOOLS IN ISOKA DISTRICT"

The University of Zambia Humanities and Social Sciences Research Ethics Committee IRB has approved the study noting that there are no ethical concerns.

On behalf of The University of Zambia Humanities and Social Sciences Research Ethics Committee IRB, we would like to wish you all the success as you carry out your study.

In future ensure that you submit an application for ethical approval early enough.

Yours faithfully,

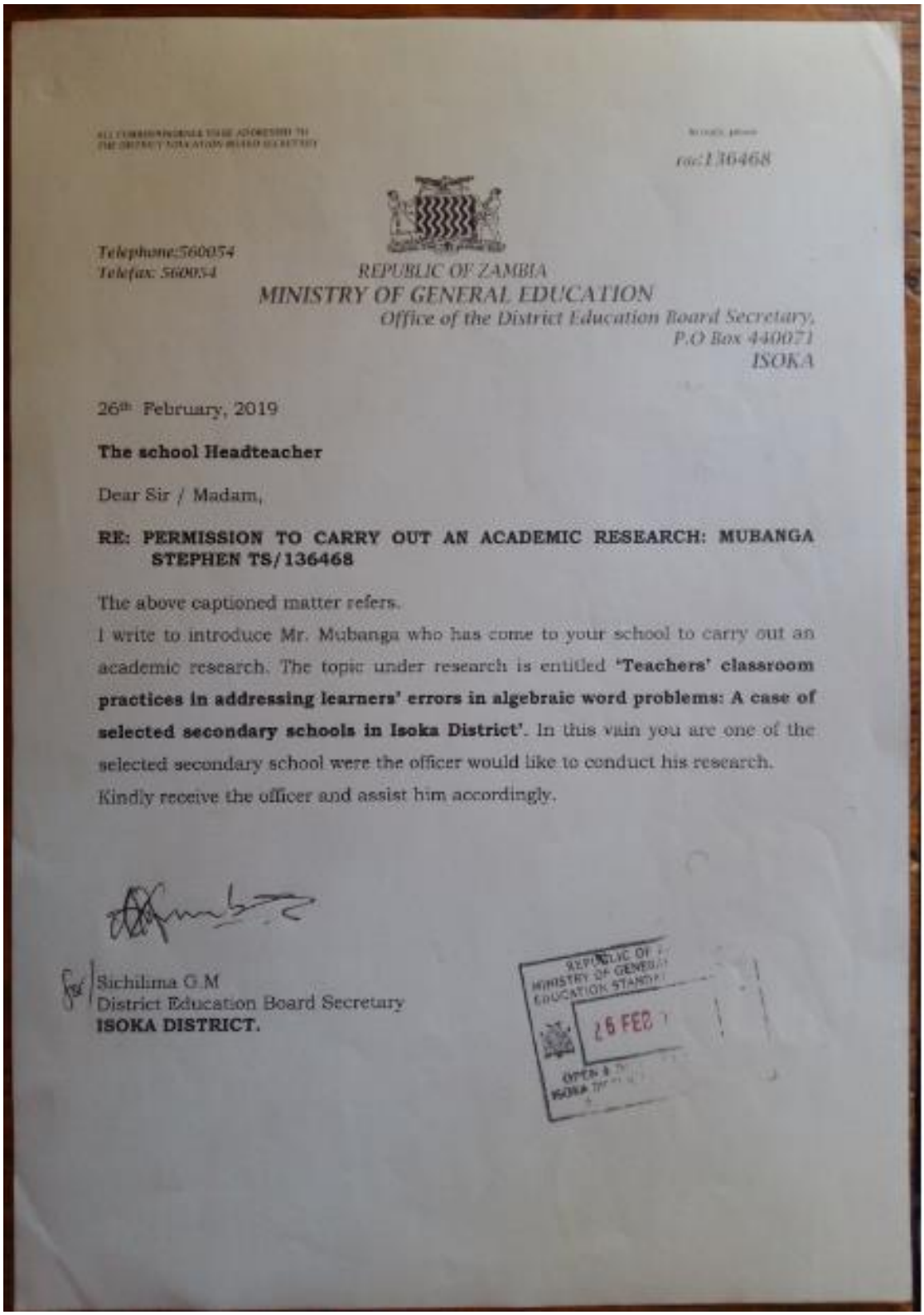
Dr. J. Mwanza
BA, MSoc, Sc., PhD
CHAIRPERSON
THE UNIVERSITY OF ZAMBIA HUMANITIES AND SOCIAL SCIENCES RESEARCH ETHICS COMMITTEE IRB

cc: Director Directorate of Research and Graduate Studies
Assistant Director (Research), Directorate of Research and Graduate Studies
Assistant Registrar (Research), Directorate of Research and Graduate Studies
Senior Administrative Officer (Research), Directorate of Research and Graduate Studies

Excellence in Teaching, Research and Community Service

APPENDIX B

Permission letter for final data collection – District Education Board Secretary (DEBS), Isoka District



ALL CORRESPONDENCE TO BE ADDRESSED TO THE DISTRICT EDUCATION BOARD SECRETARY

Please quote

ref:136468

Telephone:560054
Telefax: 560054

REPUBLIC OF ZAMBIA
MINISTRY OF GENERAL EDUCATION
Office of the District Education Board Secretary,
P.O Box 440071
ISOKA

26th February, 2019

The school Headteacher

Dear Sir / Madam,

RE: PERMISSION TO CARRY OUT AN ACADEMIC RESEARCH: MUBANGA STEPHEN TS/136468

The above captioned matter refers.

I write to introduce Mr. Mubanga who has come to your school to carry out an academic research. The topic under research is entitled **'Teachers' classroom practices in addressing learners' errors in algebraic word problems: A case of selected secondary schools in Isoka District'**. In this vain you are one of the selected secondary school were the officer would like to conduct his research.

Kindly receive the officer and assist him accordingly.

Sichilima G.M
District Education Board Secretary
ISOKA DISTRICT.

REPUBLIC OF ZAMBIA
MINISTRY OF GENERAL EDUCATION STANDARDS
26 FEB
OFFICE OF THE DEBS
ISOKA

APPENDIX C

Lesson observation protocol for teachers

Dear Participants,

I am a postgraduate student at the University of Zambia (UNZA) pursuing a Master of Education in Mathematics Education degree. I am conducting a research on the topic, 'Teachers' classroom practices in addressing learners' errors in algebraic word problems: A case of selected secondary schools in Isoka district.' You are kindly requested to feel free as I observe your lesson. The benefits of taking part in this study are that; it will enable us to share ideas on teaching algebraic word problems and it may also enhance your teaching of algebraic word problems. Be assured that your identity and the information obtained here will be confidential and will not be disclosed to anyone outside this study. You are free to withdraw from participating in the study if you feel you can no longer continue participating. All observations recorded will be taken as strictly classified. This study is purely for academic purposes and not meant to cause you any mental or physical harm. I therefore, encourage you to be free when teaching as I observe the lesson.

School code:……. Teacher's code:……

Grade:……..

Time:……..Number of Learners: Boys……. Girls……

Date:……

Observation sheet guide for teachers

S/NO	CRITERION FOR ERROR CAUSES	SUB CATEGORIES	ACTION/ EVIDENCE	COMMENT
1.	Giving learners chance to read algebraic word problems. (Reading ability).	Word recognition		
		Symbol recognition		

2.	Giving opportunities for learner to explain the meaning of algebraic word problems in their own words. (Comprehension ability).	Literal understanding		
		Specific terminology understanding		
3.	Probing learners' explanations of what they are required to do in order to get the correct answer to the algebraic word problem at hand. (Transformation ability)	Translation of question		
		Selection of mathematical processes		
4.	Paying attention to how learners execute the plans they highlighted of how the algebraic word problem must be answered. (Process skills)	Arithmetic (Numerical operations) Random response Wrong operation Faulty algorithm		

		Faulty computation No response		
		Spatial (question understood but pupil has no spatial ability and knowledge to obtain correct answer)		
		Logic (Learner fail to carry out logical thinking required to solve the problem despite understanding it)		
5.	Paying attention to the way the learners write the final answer and how they express it according to the demands of the algebraic word problem at hand. (Encoding ability)	Words (Spellings)		
		Symbols (How they are used)		

GENERAL OBSERVATION

……..…..

……

APPENDIX D

Semi-structured interview guide for teachers

Dear Participant,

I am a postgraduate student at the University of Zambia (UNZA) pursuing a Master of Education in Mathematics Education degree. I am conducting a research on the topic, 'Teachers' classroom practices in addressing learners' errors in algebraic word problems: A case of selected secondary schools in Isoka district.' You are kindly requested to participate in this research by responding to the questions below. The benefits of taking part in this study are that; it will enable us to share ideas on teaching algebraic word problems and it may also enhance your teaching of algebraic word problems. All responses recorded will be taken as strictly classified. Be assured that your identity and the information obtained here will be confidential and will not be disclosed to anyone outside this study. You are free to withdraw from participating in the study if you feel you can no longer continue participating. This study is purely for academic purposes and not meant to cause you any mental or physical harm. I therefore, encourage you to be free when providing your responses.

QUESTIONS

1. In your view, what kind of knowledge is required for learners to solve algebraic word problems?

2. In your view, what's the most significant problem solving difficulty for learners?

3. In your view, do you think algebraic word problems are difficult for learners? Give reasons.

4. Which of the learners' drawbacks make them unable to solve algebraic word problems?

5. What kind of practice do you use to reinforce learners' skills in solving algebraic word problems?

6. What kind of strategy have you used so far to teach algebraic word problems?

i. Which of the strategies do you teach to your learners?

ii. Which of the strategies is not your favourite strategy for solving algebraic word problems?

7. How do you ensure that learners become competent in solving algebraic word problems?

Thank you very much for your cooperation

APPENDIX E

Document analysis sheet for teachers

Dear Participants,

I am a postgraduate student at the University of Zambia (UNZA) pursuing a Master of Education in Mathematics Education degree. I am conducting a research on the topic, 'Teachers' classroom practices in addressing learners' errors in algebraic word problems: A case of selected secondary schools in Isoka district.' You are kindly requested to participate in this research by providing me with your lesson plans and schemes of work. The benefits of taking part in this study are that; it will enable us to share ideas on teaching algebraic word problems and it may also enhance your teaching of algebraic word problems. Be assured that your identity and the information obtained here will be confidential and will not be disclosed to anyone outside this study. You are free to withdraw from participating in the study if you feel you can no longer continue participating. All observations recorded will be taken as strictly classified. This study is purely for academic purposes and not meant to cause you any mental or physical harm. I therefore, encourage you to be free as I conduct document analysis.

School code:……. Teacher's code:……

Grade:…… Time:…….. Date:……

Number of Learners: Boys……. Girls…….

Document type:……...

Document analysis guide

S/NO	CRITERION FOR ERROR CAUSES	SUB CATEGORIES	ACTION/ EVIDENCE	COMMENT
1.	Document reflecting giving learners chance to read algebraic word problems. (Reading ability).	Word recognition		
		Symbol recognition		
		Literal understanding		

2.	Planning showing opportunities for learner to explain the meaning of algebraic word problems in their own words. (Comprehension ability).			
		Specific terminology understanding		
3.	Planning on document reflecting probing learners' explanations of what they are required to do in order to get the correct answer to the algebraic word problem at hand. (Transformation ability)	Translation of question		
		Selection of mathematical processes		
4.	The plan taking into account paying attention to how learners execute the	Arithmetic (Numerical operations) Random response Wrong operation Faulty algorithm		

	plans they highlighted of how the algebraic word problem must be answered. (Process skills)	Faulty computation No response		
		Spatial (question understood but pupil has no spatial ability and knowledge to obtain correct answer)		
		Logic (Learner fail to carry out logical thinking required to solve the problem despite understanding it)		
5.	Document reflecting teacher paying attention to the way the learners write the final answer and how they express it according to the demands of the algebraic word problem at hand. (Encoding ability)	Words (Spellings)		
		Symbols (How they are used)		

GENERAL OBSERVATION

…….

…..……...

APPENDIX F

Focus group discussion guide for learners

Dear Participants,

I am a postgraduate student at the University of Zambia (UNZA) pursuing a Master of Education in Mathematics Education degree. I am conducting a research on the topic, 'Teachers' classroom practices in addressing learners' errors in algebraic word problems: A case of selected secondary schools in Isoka district.' You are kindly requested to participate in this research by responding to the questions below. The benefits of taking part in this study are that; it will enable us to share ideas on how best you can learn algebraic word problems and it may also enhance your understanding of algebraic word problems. Be assured that your identity and the information obtained here will be confidential and will not be disclosed to anyone outside this study. You are free to withdraw from participating in the study if you feel you can no longer continue participating. All observations recorded will be taken as strictly classified. This study is purely for academic purposes and not meant to cause you any mental or physical harm. I therefore, encourage you to be free when providing your responses.

QUESTIONS

1. In your view, what kind of knowledge is required for you to solve algebraic word problems?

2. In your view, what difficulties do you experience in solving algebraic word problems? *Why do you think you face such difficulties?*

3. What measures does your teacher take to reinforce your skills in solving algebraic word problems?

4. What does your teacher do to ensure that you know what the question requires you to do before you answer it?

    (i) How can you describe his/her approaches when teaching algebraic word problems?

    (ii) How has his/her approaches affected your solving of algebraic word problems?

5. What do you think your teacher should do in order for you to solve algebraic word problems easily?

6. What do you like about your teachers' approaches when teaching algebraic word problems and why?

7. What practices would you like your teacher to improve on when teaching algebraic word problems in order for you to understand and solve the questions easily?

Thank you very much for your cooperation

APPENDIX G

Participant's consent form

Dear Participant,

I am a second year Master of Education in Mathematics Education student at the University of Zambia (UNZA) under the supervision of Dr. Nalube, P.P. I wish to conduct a research study as a final year student, which examines 'Teachers' classroom practices in addressing learners' errors in algebraic word problems.' The purpose of this study is to find out if teachers' classroom practices are aimed at identifying learners' errors in solving algebraic word problems. This will help suggest to schools teachers' classroom practices that could minimize such errors in learners. I wish to observe you teaching algebraic word problems to grade 8s first, then interview you, conduct a document analysis by looking at your schemes of work and lesson plan, and finally wish to conduct focus group discussions with the grade 8 learners. This is aimed at having depth understanding of learners' thinking process and reasons for their errors as linked to teachers' classroom practices. The interview will take about 30 minutes.

I would like to request your participation in this study. Your participation in this study is voluntary and any information collected will be kept anonymous. You are free to withdraw from participating in the study if you feel you can no longer continue. The benefits of your participation in this study is that it may improve the teaching and learning of algebraic word problems which may improve performance in mathematics.

Please indicate by filling in the form below if you accept to take part in this study. Your cooperation will be highly appreciated.

For further enquires please contact cell phone(s) 0979223034 / 0967223034 or email 2017014390@student.unza.zm

Thanks for your cooperation

Yours Sincerely,

Mubanga Stephen

I…….. Agree / disagree to participate in the study,

INTERVIEW [YES / NO] (TICK ONE)

SIGNATURE…….....

DATE ……...

Thank you